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FRAGMENTATION HIERARCHY OF BRIGHT SUNGRAZING COMETS AND THE BIRTH AND ORBITAL EVOLUTION OF THE KREUTZ SYSTEM. II. THE CASE FOR CASCADING FRAGMENTATION

Zdenek Sekanina and Paul W. Chodas
Jet Propulsion Laboratory, California Institute of Technology, Pasadena, CA 91109; zs@sek.jpl.nasa.gov, paul.w.chodas@jpl.nasa.gov
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ABSTRACT

We examine the process of cascading fragmentation for the Kreutz sungrazer system to continue our exploration of its birth, orbital evolution, and temporal clumping. We modify and broaden the two-superfragment model from Paper I to include clusters of ~30 bright comets spanning four centuries and 1000 *SOHO* sungrazers from 1996 to 2006. The spectacular parent sungrazer X/1106 C1 is assumed to have tidally split shortly after perihelion into a train of major protofragments immersed in a cloud of particulate debris, which at larger heliocentric distances were breaking up nontidally over and over again. We describe potential evolutionary paths for the Kreutz system by linking X/1106 C1 in subgroup I–type orbit with the comet of February 423 in one scenario or with the comet of February 467 in another. The latter scenario accounts for sungrazer clusters in as early as the 16th century, suggests that the progenitor object may have been observed as the comet of 214 BCE, is quite consistent with the orbital distribution of the *SOHO* sungrazers that sample the central filament of the Kreutz system between the clusters of major sungrazers, and predicts future clusters until ~2120. Comet X/1106 C1 and the common parent of C/1882 R1 and C/1965 S1 were two first-generation fragments of the progenitor that split nontidally on the way to its 5th century perihelion, reminiscent of the superfragments in Paper I. We provide computational tools needed for solving the problem of the Kreutz system's orbital evolution, but no unique scenarios are presented for the individual comets. Another cluster of bright sungrazers is expected to arrive in the coming decades, its earliest member possibly just several years from now.

Subject headings: comets: general — methods: data analysis

1. INTRODUCTION

A recently developed two-superfragment model for the birth and evolution of the Kreutz system (Sekanina & Chodas 2004, hereafter Paper I) presents a self-consistent pyramidical construct describing the fragmentation hierarchy of eight bright sungrazers discovered between 1843 and 1970. They have their perihelia located within 1 solar radius ($1\ R_\odot = 0.0046524$ AU) of the Sun's photosphere, and all are found to exist as separate objects for less than 1700 yr, some of them for less than 300 yr.

Although it had been suggested that these bright members of the Kreutz system discriminate into two distinct subgroups (Hasegawa 1966; Kresák 1966), a major result of Paper I was the finding that a sungrazer can easily transit from one subgroup to the other because of the extra momentum it acquires during fragmentation events experienced in the course of a single revolution about the Sun. Accordingly, the subgroups do not have profound evolutionary ramifications, contrary to their traditional portrayal.

The two-superfragment model explains the origin of the eight major sungrazers as products of a small number of nontidal fragmentation events, involving separation velocities of up to 10 m s^{-1}. Since observations of comets C/1882 R1 and C/1965 S1 imply that tidally driven splitting also plays a major role in the orbital evolution of the Kreutz system and since the separation velocity range derived from the orbital distribution of nucleus fragments of C/1882 R1 (§ 3) does not generally exceed ~5 m s^{-1}, we search for ways to accommodate these constraints in the proposed fragmentation scenarios that not only incorporate most attributes of the two-superfragment model but also broaden the scope of investigation, opening an avenue for describing the evolution of other members of the Kreutz system, including the large population of minisungrazers (§ 2.1).

2. RESEARCH OBJECTIVES

If the role of the subgroups is not dynamically dominant, we must ask, what observational evidence could provide the clue to the Kreutz system origin? In the following we focus on a prominent feature of the distribution of bright sungrazers: their long-term clustering. We show that, significantly, the topic expansion from the bright sungrazers to the entire Kreutz system is inherently related to this phenomenon.

2.1. *Clustering and Tidally Driven Splitting*

Clustering of the bright sungrazers with time has long been recognized (e.g., Kreutz 1901; Marsden 1967; Hasegawa & Nakano 2001; Strom 2002). Three of the eight bright comets of the 19th and 20th centuries known to have made up the Kreutz system before 1979 had arrived in 1880–1887 (C/1880 C1, C/1882 R1, and C/1887 B1), and another three in 1963–1970 (C/1963 R1, C/1965 S1, and C/1970 K1). Although this remarkable distribution cannot possibly be fortuitous, there is no correlation between the cluster members and the subgroup members: the first and last comets from the 19th century compact cluster belong to subgroup I and the middle to subgroup II, whereas the sungrazers from the 20th century cluster belong, respectively, to subgroups I, II, and IIa (Marsden 1989).

If this clumping is indeed a dynamically important discriminator, meaning that, in general, the sungrazers in one cluster are more closely related to one another than to the sungrazers in the other cluster, the temporal separation of the clusters, 80–90 yr, could indicate a difference between the orbital periods of two *protofragments* of a *parent* sungrazer that might be identical with X/1106 C1, a spectacular object recorded in numerous historical sources and discussed many times in the past in connection with the Kreutz system (e.g., Kreutz 1888, 1901; Marsden 1967, 1989; Hasegawa & Nakano 2001; Paper I). This parent sungrazer is not necessarily identical with the Kreutz system's *progenitor*; rather, it could be the progenitor's first-generation fragment. The parent's protofragments are then second-generation fragments of the progenitor.

It is known that the sungrazers C/1882 R1 and C/1965 S1 almost certainly split from a shared parental object at the beginning of the 12th century (Marsden 1967; Sekanina & Chodas 2002a), even though they belong to different clusters. If the orbit of X/1106 C1 was that of subgroup II (as assumed in Paper I), the 1882–1965 pair can be understood as a product of a secondary, posttidal breakup (Sekanina & Chodas 2002a). We show in § 8 that the pair's age greater than one orbital revolution is in fact ruled out because the clustering effect disappears after the very first return of fragments to the Sun. If the orbit of X/1106 C1 is that of subgroup I (§ 6.4), C/1882 R1 and C/1965 S1 would be born from another parent that would pass perihelion at nearly the same time as X/1106 C1. Both X/1106 C1 and this parent would have been the first-generation products of the progenitor's nontidal breakup at large heliocentric distance during an earlier revolution about the Sun. This is of course a variation on the two-superfragment model described in Paper I, yet the relaxation of the subgroup constraint on X/1106 C1 will be shown to allow us to broaden substantially the scope of our investigation. The rest of the dynamical evolution is fundamentally independent of the subgroup type of the orbit of X/1106 C1.

It must be emphasized that an orbital period difference of somewhat less than 100 yr between two neighboring fragments of a Kreutz sungrazer is the only major orbital effect resulting from a tidally driven, near-perihelion splitting with a separation velocity of a few meters per second (e.g., Sekanina 2002a). Diagnostic information of this kind is examined in § 3 for the tidally split sungrazer C/1882 R1. Although in practice the situation is more complex, as discussed further in this paper, the clusters of bright sungrazers separated from one another by the less than 100 yr intervals represent in principle a basic attribute of the fragmentation hierarchy that determines the temporal distribution of members of the Kreutz system; each protofragment becomes the building block of a cluster, and its subsequent nontidal splitting (§ 2.2), usually very far from the Sun, gives birth first to *precursor* objects (or third-generation fragments of the progenitor) and, through them, eventually to members of a cluster, which are fourth- and/or higher generation fragments.

With X/1106 C1 as the parent comet, the orbital period of the protofragment responsible for the core of the 19th century cluster was between 774 and 781 yr, whereas the period of the protofragment responsible for the core of the 20th century cluster was between 857 and 864 yr. This scenario is supported by Strom's (2002) recent finding that there is evidence for clusters of sungrazers apparently detected by the Chinese in broad daylight during the 18th, 17th, and even 16th centuries. A strongly nonrandom cluster-like temporal distribution of possible sungrazers in the historical records was also suggested by Hasegawa & Nakano (2001). By extension, one may expect that clusters of bright, closely spaced Kreutz system comets will arrive during the 21st century (for a further discussion see § 6.4) and perhaps beyond. Thus, there apparently were more than two protofragments.

A tendency to clumping is not a privilege of only the bright sungrazers. Minor members of the Kreutz system cluster on much shorter timescales. Marsden (1989) commented on two pairs with a temporal separation of less than 2 weeks among the sungrazers detected with the *Solar Maximum Mission* (*SMM*) coronagraph in the 1980s. A high degree of clustering among 1000 minisungrazers, discovered more recently with the coronagraphs on board the *Solar and Heliospheric Observatory* (*SOHO*), many of which arrived only a small fraction of a day apart, has been well known and is fully understood (Sekanina 2002b).

2.2. *Nontidal, Secondary Fragmentation*

Besides the tidally driven splitting, the proposed scenario requires secondary fragmentation events to explain (1) the distribution and orbital diversity of the sungrazers both in and outside the compact cores of the 19th and 20th century clusters and (2) the populations of fainter Solwind, *SMM*, and *SOHO* sungrazers, discovered coronagraphically since 1979. The respectable number of these minor objects, almost all of which were episodically, throughout one revolution about the Sun, generated from the same protofragments as the bright sungrazers (Sekanina 2002a), provides ultimate evidence on the process of *cascading* fragmentation. Only some of the coronagraphically discovered minicomets, which move in orbits that, except for the perihelion time, are nearly identical with the parent comet's orbit, could represent chips of leftover material from the tidally driven events that gave birth to these protofragments in the early 12th century (§ 6.4).

The mass distribution of Kreutz comets as products of the process of cascading fragmentation has recently been investigated by Sekanina (2003), who showed that the rate of the *SOHO* sungrazers is governed by a power law, whose cumulative distribution indicates that at least 50% (and possibly much more) of the total mass is locked in the largest fragment. It is of course the objects at the upper end of the mass spectrum that have the best chance of being detected from the ground as bright sungrazers, unless their arrival occurs between mid-May and mid-August, the period of unfavorable observing conditions (daylight detections only).

2.3. *Fragmentation Sequence and Hierarchy*

The now defunct comet D/1993 F2 (Shoemaker-Levy 9) provides a compelling example of the process of cascading fragmentation, in which the primary, tidally driven breakup of a parent into several protofragments was followed by numerous episodes of their secondary, nontidal splitting into smaller fragments, some of which later disappeared (Sekanina et al. 1998). Eventually, by the time of the comet's collision with Jupiter, about two dozen nucleus fragments (condensations) were observed to line up in a train, which was described by several observers as a "string of pearls" (Fig. 1). The established fragmentation sequence provides an important analog for the evolution of the Kreutz system, albeit on a very different timescale. Three important properties of the image in Figure 1 are noted: (1) the brightest condensations tend to be located near the middle of the train; (2) the condensations are nearly, but not perfectly, equidistant; and, very significantly, (3) the space between the condensations is filled with a large amount of diffuse material, which makes up a sheath connecting and encompassing the pearls and which represents the population of fragments too small to detect individually.

Comparing Figure 1 to the Kreutz system, the analogy to comet D/1993 F2 becomes obvious: the pearls correspond to the cores of clusters of the bright sungrazers, while a small segment of the sheath of material spread in between two particular pearls has been sampled thanks to the discovery of the *SOHO* and other minisungrazers. Because of the favorable circumstances and the advanced techniques employed, we have so far been able to detect over 1000 individual members of this debris population.

More recently, the cascading nature was independently illustrated for the fragmentation process of the Marsden and Kracht

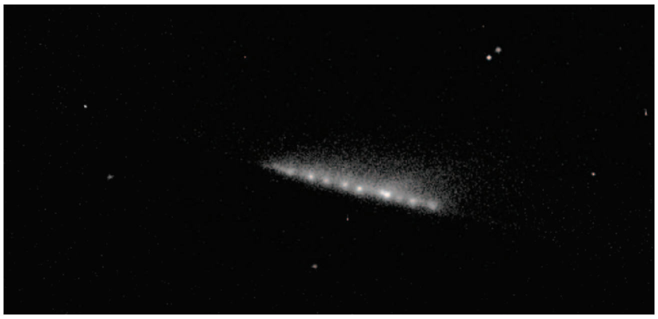

Fig. 1.—Image of D/1993 F2 (Shoemaker-Levy 9) taken by M. Lindgren at the European Southern Observatory on 1993 March 28. The nuclear train, also called a "string of pearls," is ~50″ long; the Sun is to the lower left. Note that the train is immersed in an elongated sheath of diffuse particulate material, whose brightness falls off in the direction perpendicular to the train and with increasing distance from each condensation. (Courtesy of European Southern Observatory.)

groups of sunskirting comets belonging to the Machholz interplanetary complex (Sekanina & Chodas 2005) and, in a spectacular fashion, by comet 73P/Schwassmann-Wachmann, whose orbital period is about 5.4 yr. With its perihelion distance of nearly 1 AU, the comet cannot undergo close approaches to the Sun or Jupiter and cannot be exposed to substantial tidal forces. The known history of fragmentation events of comet 73P dates back two revolutions about the Sun to late 1995, when a multiple nucleus was first reported (Boehnhardt & Käufl 1995; Boehnhardt et al. 1996). The comet's pre-2006 behavior was investigated and described by Sekanina (2005), who explicitly predicted the possibility of detection of "a number of fainter fragments distributed along the orbit" during the comet's exceptionally favorable 2006 return, which included a fairly close encounter with Earth. Indeed, more than 60 reported fragments were assigned official designations by the Minor Planet Center, with an unofficial fragment count in the hundreds. Fascinating images of more than 30 larger fragments, many with parallel tails, were taken in the infrared light with the *Spitzer Space Telescope* (Vaubaillon & Reach 2006). The case of comet 73P demonstrates that the process of cascading fragmentation can take place even in the absence of tidal breakup, but the split comet's appearance is then different in that the brightest condensation is the leading fragment and the separations between the condensations are completely nonuniform: the first two characteristics of tidally driven fragmentation are thus missing (cf. Sekanina 1997). However, the space between the condensations of 73P was filled with diffuse material, so that the third characteristic of the image in Figure 1 is independent of whether or not tidally driven fragmentation is involved. The case of comet 73P greatly supports the notion that cascading fragmentation of comets is an omnipresent process, which can be examined in detail on those rather rare occasions when favorable conditions allow a large number of diagnostic observations to be acquired.

Two early stages of the tidally driven cascading fragmentation process are shown schematically in Figure 2. The top left panel displays a parent comet just after its perihelion passage and very shortly before its tidally driven splitting. The outcome is exhibited in the top right panel. With their separations greatly exaggerated, six protofragments are depicted shortly after the fragmentation event. They are lined up according to their relative velocities acquired at breakup. The actual number of protofragments is of course given by the number of sungrazer clusters. The birth of all protofragments is not necessarily simultaneous (§ 3), as the process of tidally driven splitting may consist of several separate events, spanning perhaps a few hours or so.

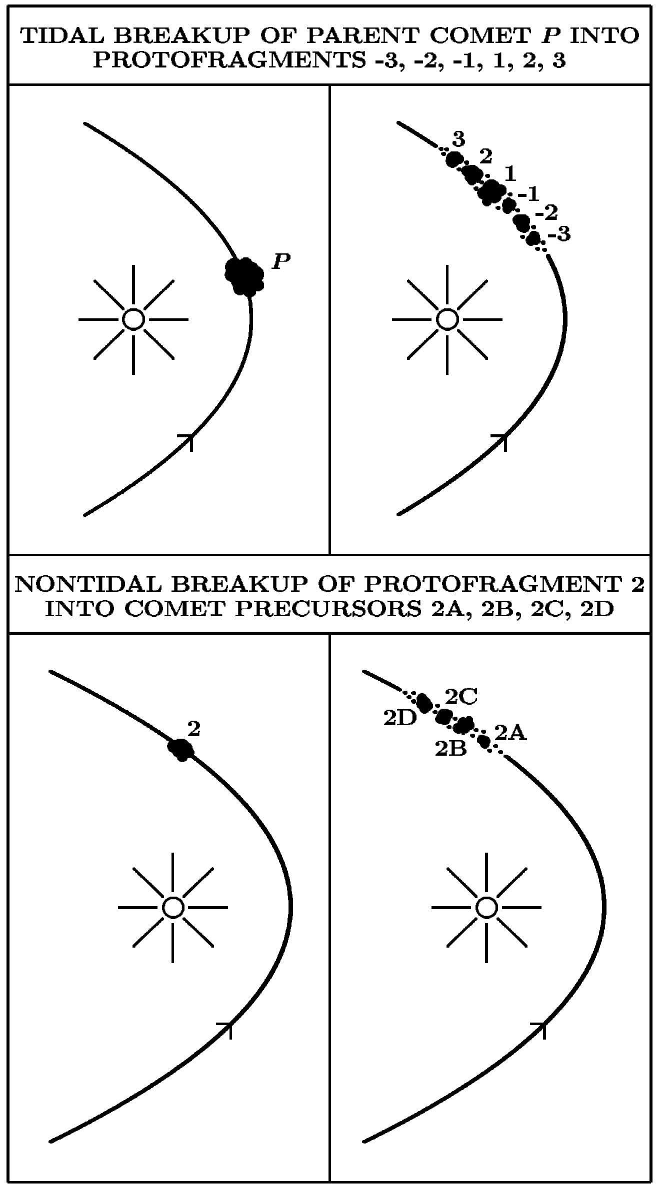

Fig. 2.—First two stages of the cascading fragmentation process for the Kreutz system (schematically). *Top*: Parent comet just after its 1106 perihelion passage and shortly before its tidally driven splitting (*left*), and six protofragments (with their separations greatly exaggerated) and a sheath of particulate material after the parent comet's breakup (*right*). *Bottom*: Protofragment 2 shortly before its nontidal splitting (*left*), and four comet precursors (with their separations greatly exaggerated) and a sheath of particulate material after the breakup of protofragment 2 (*right*). Protofragments can split nontidally at any orbital location.

The protofragments are in Figure 2 numbered in the chronological order of their next passage through perihelion. If protofragment 1 is the source of the cluster in the 1880s, protofragment 2 could in principle be the source of the 20th century cluster, protofragment 3 should refer to a future cluster expected to arrive during the 21st century (§§ 2.1 and 6.4; also Paper I), whereas protofragments −1, −2, and −3 could be the birthplaces of clusters that had returned to the Sun before the 1880s.

The lower half of Figure 2 depicts schematically the breakup of protofragment 2 into precursors 2A, 2B, . . . , which could represent the parent bodies of the sungrazers of the 20th century cluster. While the impression from the figure may be that a protofragment breaks into precursors simultaneously and soon after its own birth, this is not in fact the case. The formation of precursors is a stochastic fragmentation process, for which the orbital locations of breakup events vary not only from protofragment to protofragment but also from precursor to precursor for any given protofragment. A protofragment's splitting may occur days, months, years, or even centuries after the parent's breakup. The heliocentric distances involved range from tenths of 1 AU to and past aphelion. Separation velocities of a few meters per second at distances beyond ∼2 AU from the Sun can account for perihelion arrival times of up to a few tens of years apart (Table 8 of Sekanina 2002a), thus explaining the temporal distribution of sungrazers within a cluster.

Each precursor continues to fragment episodically into smaller pieces at larger heliocentric distances, where the separation velocity affects the orbital motions of fragments very differently (Table 9 of Sekanina 2002a; Table 4 of Sekanina & Chodas 2002b). At distances exceeding 100 AU, the acquired extra momentum causes only a minor perturbation of the orbital period, thus accounting for fragments subsequently arriving at the Sun nearly simultaneously, but exerts a major effect on both the angular orbital elements and perihelion distance, thus accounting for the orientation diversity of the orbital planes and perihelion distances of fragments. Because of the mass distribution of the fragmentation products, there are eventually only a few survivors massive enough to be detected as bright members of the Kreutz system (§ 2.2).

The cascading model does not require any excessive mass to explain the enormous population of the Kreutz system. A collection of the *SOHO*-like sungrazers evenly distributed along the entire orbit about the Sun was estimated by Sekanina (2002a) at 200,000, with its total mass comparable to that of a sphere about 1 km across. The mass of the progenitor comet is strongly dependent on the number and mass of the parents and protofragments it split into, an issue addressed in § 8. Our current estimates suggest that the bright sungrazers are most probably the progenitor's fourth- or fifth-generation fragments. Fragments of higher generations are unlikely to be massive enough to account for the observed prominence of the bright sungrazers.

Finally, there is a question of possible earlier Kreutz systems, whose birth would have been the result of an earlier sequence of fragmentation events during the progenitor's returns to the Sun. While we cannot offer any compelling information on this issue, a few remarks are in § 8.

3. LESSONS LEARNED FROM C/1882 R1

Comet C/1882 R1, unquestionably the most extensively observed sungrazer ever and by far the brightest during the past 200 years, was reported to have displayed up to six separate nuclear condensations starting a little less than 2 weeks after its perihelion passage (Kreutz 1888). By contrast, the comet had always exhibited a single nucleus condensation before perihelion and up to 10 days after perihelion (albeit somewhat elliptical at the end), as described in detail by Gill (1883). Because of the alignment of the fragment condensations in a train embedded in an elongated sheath of diffuse material, they were sometimes likened by the observers—just as the condensations of D/Shoemaker-Levy 9 more than 100 years later (Fig. 1; § 2.3)—to a "string of pearls" or "beads on a thread of worsted."

The direction in which the nuclear fragments of C/1882 R1 were distributed in projection onto the plane of the sky never differed by more than about 30° from the Sun's projected direction. With increasing heliocentric distance, the nuclei were called 1, 2, etc. by Kreutz (1888). We use letters A = 1, B = 2, etc. in this paper, consistent with Sekanina & Chodas (2002a).

TABLE 1

Distribution of Osculation Orbital Periods among Nuclei of Comet C/1882 R1 (Epoch 1882 September 20.96 ET)

	Set of Orbital Elements[a]					
	Newtonian			Relativistic		
Point in Nuclear Train	$P_{\rm osc}$ (yr)	$\widehat{\Delta P}_{\rm osc}$ (yr)		$P_{\rm osc}$ (yr)	$\widehat{\Delta P}_{\rm osc}$ (yr)	
Condensation A	664.3			659.0		
		104.9			100.5	
Condensation B	769.2			759.5		
		48.1			43.8	
Center of mass	817.3		106.0	803.3		103.2
		57.9			59.4	
Condensation C	875.2			862.7		
		84.2			80.2	
Condensation D	959.4			942.9		

[a] Probable orbital period errors as listed by Kreutz (1891) are ±2 to ±6 yr.

Kreutz (1891) calculated several sets of orbital elements for four of the six nucleus components: A, B, C, and D. Only for the two most persistent fragments, B and C, was he able to determine the orbital sets based on the postsplit astrometric observations alone. For A–D he derived orbits by linking the astrometric observations for each of them with those for the single nucleus before its breakup. Although dynamically questionable, this is a pragmatic approach that is often used to reduce uncertainties of orbital solutions for components of split comets. Then he derived alternative orbits on the assumption that the four condensations shared a common orbital plane. He next assumed that the breakup was caused by an internal "perturbing force" pointing in a direction of (or opposite) the orbital velocity vector. Using Encke's (1823) paradigm, Kreutz (1891) postulated that the magnitude of the force varied as the square of the comet's orbital velocity and inversely as the square of heliocentric distance, with the comet's center of mass, midway between B and C, unaffected. Kreutz's solutions based on this concept yielded his final orbital sets.

In Table 1 the osculation orbital periods from these orbital sets (Kreutz 1891) are referred to as Newtonian. The probable errors involved are, according to Kreutz, between ±2 and ±6 yr. The orbital periods in the last column include the relativistic effect and were derived by comparing Kreutz's Newtonian solution for nucleus B with Hufnagel's (1919) relativistic solution. The orbital set for the center of mass of C/1882 R1, derived in Sekanina & Chodas (2002a) and presented in Table 2, gives a period nearly identical with that in Table 1. Astonishingly, the differences $\widehat{\Delta P}_{\rm osc}$ show that the orbital periods of the neighboring fragments increase with their increasing heliocentric distance by 80–100 yr, thus matching closely the temporal separation between the two clusters of bright sungrazers in the 19th and the 20th centuries. We conclude that in this sense *the nucleus condensations are related to the presplit nucleus of comet C/1882 R1 the same way as the protofragments—the building blocks for the sungrazer clusters—are related to their parent*. To our knowledge, this remarkable coincidence has never been noticed before.

An increment of ∼80 yr also shows strikingly in the distribution of perihelion times predicted for the next returns of nucleus condensations A–D of C/1882 R1, when integrating their motions forward in time. The results of this numerical exercise, discussed in some detail in § 8, indicate that, if subjected to no further fragmentation, the four pieces would arrive at the Sun in the 25th

TABLE 2
Adopted Orbital Elements for Comet C/1882 R1 (Equinox J2000.0)

Orbital Element	Comet C/1882 R1[a]
Time of perihelion passage T (ET)	1882 Sep 17.72410 ± 0.00004
Argument of perihelion ω (deg)	69.5851 ± 0.0018
Longitude of ascending node Ω (deg)	347.6559 ± 0.0022
Orbital inclination i (deg)	142.0109 ± 0.0005
Perihelion distance q (AU)	0.0077508 ± 0.0000007
Orbital eccentricity e	0.99991034 ± 0.00000016
Orbital period P (yr)	803.7 ± 2.2
Epoch (ET)	1882 Oct 2.0

[a] Planetary perturbations and relativistic effect included (for details see Sekanina & Chodas 2002a).

to 28th centuries, over a period of more than 230 yr. On the average, the differences between the predicted arrival times of two neighboring condensations are about 17 yr shorter than their near-perihelion osculating values in Table 1. For the first two pairs the differences in the arrival times are virtually identical with the mean separation of 83 yr between the 19th and 20th century clusters. Since fragmentation of C/1882 R1 will in all probability continue during the intervening time, we predict that five to seven centuries from now observers will have a chance to witness new sungrazer clusters, each approximately centered on the predicted arrival times.

It is known that differential momenta, equivalent to separation velocities of 5 m s^{-1}, acquired by fragments on their release from a parent sungrazer at or very close to perihelion (tidally driven events), affect the orbital periods by a factor ranging from $\sim\frac{1}{2}$ to several (Sekanina 2002a); a sungrazer whose orbital period was initially ∼800 yr can at perihelion release fragments into orbits with periods in a range from ∼400 to ∼3000 yr, given this separation velocity. This is so because the sungrazers orbit in extremely elongated ellipses that deviate only slightly from a parabola near perihelion. Accordingly, their orbital velocity is then almost equal to the velocity of escape from the Sun and even a minute change in the momentum can represent a large fraction of the difference between the parent's orbital velocity and the escape limit. The sense of this strongly nonlinear perturbation depends critically on the sense of the momentum change. The role of tidally driven splitting in the immediate proximity of the Sun is to scatter the fragments into orbits, which, while looking practically identical near perihelion (and having nearly identical elements ω, Ω, i, and q), bring the fragments back to the Sun at greatly diverse times. This is how the protofragments—the parents of the future sungrazer clusters—are born.

Since the effect that a separation velocity has on the orbital period is very sensitive to the time of tidally driven fragmentation, $t_{\rm frg}$, it is desirable to investigate this issue in detail (§ 4). Of particular interest is whether the event occurs before, at, or after the comet's closest approach to the perturbing body. To our knowledge, this issue has never been systematically addressed for the sungrazers.

For a given fragmentation time $t_{\rm frg}$, the change in the orbital period can be examined as a function of the fragment's momentum increment, i.e., its separation velocity vector $\boldsymbol{V}_{\rm sep}(t_{\rm frg})$ relative to the parent comet. To describe this vector, we employ the comet's orbital plane as the reference plane and introduce the components of the separation velocity in, respectively, the radial (away from the Sun), transverse, and normal directions of the right-handed RTN coordinate system, so that $\boldsymbol{V}_{\rm sep}(t_{\rm frg}) = \{V_R(t_{\rm frg}), V_T(t_{\rm frg}), V_N(t_{\rm frg})\}$. The orbital period is perturbed by the component $V^*_{\rm sep}(t_{\rm frg})$ that points in the direction of the orbital velocity vector $\boldsymbol{V}(t_{\rm frg})$. Of the infinite number of vectors $\boldsymbol{V}_{\rm sep}(t_{\rm frg})$ yielding the same value of $V^*_{\rm sep}(t_{\rm frg})$, only two special cases are considered here, for which $\boldsymbol{V}_{\rm sep}(t_{\rm frg}) = \{V_R(t_{\rm frg}), 0, 0\}$ and $\boldsymbol{V}_{\rm sep}(t_{\rm frg}) = \{0, V_T(t_{\rm frg}), 0\}$, respectively. In the first case, $V_R(t_{\rm frg})$ is equal to

$$V_R(t_{\rm frg}) = \frac{\mp V^*_{\rm sep}(t_{\rm frg})}{\sqrt{1 - (k/r_{\rm frg}V_{\rm frg})^2 q(1+e)}}, \qquad (1)$$

whereas in the second case $V_T(t_{\rm frg})$ is

$$V_T(t_{\rm frg}) = V^*_{\rm sep}(t_{\rm frg}) \frac{r_{\rm frg}V_{\rm frg}}{k\sqrt{q(1+e)}}, \qquad (2)$$

where $V_{\rm frg} = V(t_{\rm frg}) = |\boldsymbol{V}(t_{\rm frg})|$ (in km s^{-1}), q is the parent comet's perihelion distance (in AU), e is its eccentricity, $r_{\rm frg} = r(t_{\rm frg})$ is its heliocentric distance (in AU) at the time of fragmentation, and $k = 29.78$ km s^{-1} $AU^{1/2}$. In the parabolic approximation, which satisfies our needs, equations (1) and (2) simplify, respectively, to

$$V_R(t_{\rm frg}) = \frac{\mp V^*_{\rm sep}(t_{\rm frg})}{\sqrt{1 - q/r_{\rm frg}}} \qquad (3)$$

and

$$V_T(t_{\rm frg}) = V^*_{\rm sep}(t_{\rm frg})\sqrt{\frac{r_{\rm frg}}{q}}, \qquad (4)$$

where $r_{\rm frg}$ is related to $t_{\rm frg}$ (in days from the parent comet's time of perihelion passage) by

$$t_{\rm frg} = \mp K(r_{\rm frg} + 2q)\sqrt{r_{\rm frg} - q}, \qquad (5)$$

with $K = 27.404$ days $AU^{-3/2}$. The negative signs in equations (1), (3), and (5) apply to fragmentation events before perihelion, the positive signs to postperihelion episodes.

Turning now to the data on condensations A–D of comet C/1882 R1 in Table 1, the dependence of the orbital period $P_{\rm osc}$ on the separation velocity $V^*_{\rm sep}(t_{\rm frg})$ will be sought in the general form

$$P_{\rm osc} = P_0 + \sum_{j=1}^{n} A_j(t_{\rm frg})\left[V^*_{\rm sep}(t_{\rm frg})\right]^j, \qquad (6)$$

where $P_0 = 803.3$ yr is the relativistic orbital period for the comet's center of mass at an osculation epoch of 1882 September 20.96 ET. The coefficients $A_j(t_{\rm frg})$ can be derived from a series of

TABLE 3

Separation Velocity Components for Nucleus Condensations of Comet C/1882 R1 Fitting Their Orbital Periods (Epoch 1882 September 20.96 ET)

		Separation Velocity Components[a] (m s^{-1}) for Fragmentation Time						
		0.1 days before Perihelion			At Perihelion	0.1 days after Perihelion		
Nuclear Condensation	Orbital Period[b] (yr)	V^*_{sep}	V_R	V_T	$V^*_{sep} \equiv V_T$	V^*_{sep}	V_R	V_T
A	659.0	−2.4	+3.1	−3.7	−1.5	−2.4	−3.1	−3.7
B	759.5	−0.6	+0.8	−1.0	−0.4	−0.6	−0.8	−1.0
C	862.7	+0.8	−1.0	+1.2	+0.5	+0.8	+1.0	+1.2
D	942.9	+1.7	−2.2	+2.7	+1.1	+1.7	+2.2	+2.7

[a] Relative to the comet's center of mass: V^*_{sep} is the separation velocity component in the direction of the orbital velocity vector; V_R and V_T are the equivalent velocity components in, respectively, the radial and transverse directions of the right-handed RTN coordinate system, which would yield the same perturbation of the orbital period as V^*_{sep}. Note the symmetries with respect to perihelion.

[b] From the fourth column of Table 1.

computer runs for t_{frg} = const and different values of $V^*_{sep}(t_{frg})$, using a back-and-forth orbit integration technique previously employed to develop the two-superfragment model in Paper I and described in some detail in Sekanina & Chodas (2002a). The calculations show that for a separation velocity of up to a few meters per second P_{osc} can be fitted to within ±0.1 yr when $n = 3$. For example, for $t_{frg} = +0.1$ days equation (6) reads

$$P_{osc} = 803.3 + 71.8V^*_{sep} + 5.46\left(V^*_{sep}\right)^2 + 0.37\left(V^*_{sep}\right)^3, \quad (7)$$

where P_{osc} is in yr and V^*_{sep} in m s^{-1}.

Table 3 indicates that the derived values of the separation velocity components V^*_{sep} for the nucleus condensations A–D remain below 2.5 m s^{-1} relative to the comet's center of mass as long as the fragmentation event occurs within 0.1 days of perihelion. The table also shows that the required total separation velocity V_{sep} stays below 4 m s^{-1} if its vector is perpendicular to the radius vector in the orbit plane ($V_{sep} = V_T$; $V_R = V_N = 0$). On the other hand, a higher separation velocity is implied if its vector points nearly in the radial direction [note that $V_R(t_{frg}) \rightarrow \mp\infty$ for $t_{frg} \rightarrow 0$ in eq. (3)]; it drops below 4 m s^{-1} only at fragmentation times between ∼40 minutes and 7.5 hr from perihelion for condensation A and between ∼25 minutes and 22 hr for condensation D. The required value of $V_{sep} = V_R$ reaches a minimum when the fragmentation event occurs ∼1.8 hr or ∼0.075 days from perihelion, at which time condensation A is fitted with about 3 m s^{-1} and condensation D with only 2.2 m s^{-1}. These numbers demonstrate the importance of the tidal fragmentation time, justifying a detailed study (§ 4).

Based on an assumed orbital identity of comets X/1106 C1 and C/1882 R1, Figure 3 shows a plot of separation velocities needed for fragmentation events occurring between 5 hr before and 5 hr after the 1106 perihelion time to make a fragment return to the Sun at the time of C/1965 S1, about 83 yr after comet C/1882 R1. The required separation velocity does not generally exceed 3 m s^{-1}, except within less than 0.5 hr of the perihelion passage in the case of a separation velocity pointing in the radial direction. Comparison of the results in Figure 3 and Table 3 thus illustrates the previously detected high degree of correspondence between the relation of the nuclear condensations of C/1882 R1 to its presplit nucleus on the one hand and the relation of the 19th and 20th century sungrazer clusters to the parent comet X/1106 C1 on the other hand.

4. TIME OF TIDALLY DRIVEN FRAGMENTATION

The time of tidally driven fragmentation can with fair accuracy be derived for three objects: two sungrazers, C/1882 R1 and C/1965 S1, and the defunct comet D/Shoemaker-Levy 9, for which the results were already published by Sekanina et al. (1998). The time of sudden nucleus disintegration is known for another sungrazer, C/1887 B1 (Sekanina 1984).

Here we begin with C/1882 R1, using the astrometric offsets of the nucleus condensations that were collected by Kreutz (1888, 1891, 1901). We employ a two-parameter fragmentation model (solving only for the fragmentation time and a differential deceleration; see Sekanina 1977, 1982), an approach that is demanded by the limited accuracy of the observations. Below we refine and extend the previous investigation (Sekanina 1978, 1982) to a total of six condensations, including E and F.

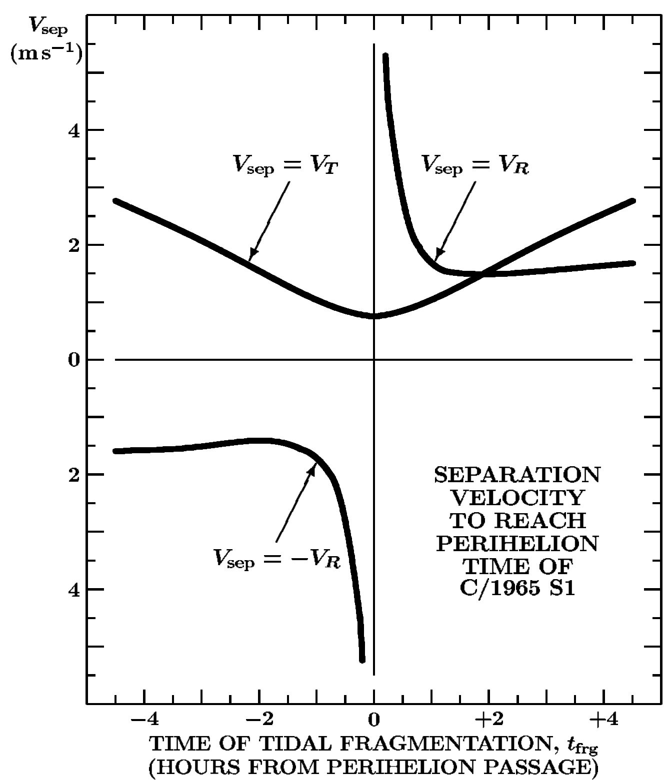

Fig. 3.—Dependence of the separation velocity on the time of tidally driven fragmentation. Assuming an orbital identity of X/1106 C1 and C/1882 R1, the plot shows the separation velocity in the radial direction ($V_{sep} = V_R$) and in the transverse direction ($V_{sep} = V_T$) needed to delay the return to the Sun from 1882 September 17 (C/1882 R1) to 1965 October 21 (C/1965 S1). Positive V_R is away from the Sun, and positive V_T is ahead of the comet in the sense of its orbital motion.

TABLE 4

Astrometric Offsets of Nucleus Condensations A, C, D, E, and F of Comet C/1882 R1 from Condensation B (Equinox B1950.0)

Date (UT)	Observed Offset[a] in R.A.[b] (arcsec)	Observed Offset[a] in Decl. (arcsec)	Residual $O-C$ in R.A.[b] (arcsec)	Residual $O-C$ in Decl. (arcsec)	Condensation	Observer and Site
1882 Oct 2.12	−9.8	+1.8	0.0	+0.9	F	Elkin (Cape of Good Hope)
1882 Oct 4.11	−12.9	+1.6	−1.3	+0.4	F	Elkin (Cape of Good Hope)[c,d]
1882 Oct 5.44	−6.1	+0.8	−1.9	+0.4	C	Wilson (Cincinnati)
1882 Oct 6.19	−16.7	+3.0	(−3.3)	(+1.3)	F	Krueger (Kiel)
1882 Oct 6.42	+6.2	−0.8	−0.3	−0.4	A	Wilson (Cincinnati)
1882 Oct 6.42	−5.5	+0.8	−1.0	+0.4	C	Wilson (Cincinnati)
1882 Oct 6.44	+5.9	−1.1	−0.6	−0.6	A	Wilson (Cincinnati)
1882 Oct 6.44	−6.2	+1.1	−1.7	+0.7	C	Wilson (Cincinnati)
1882 Oct 6.45	−5.3	+1.0	−0.8	+0.5	C	Wilson (Cincinnati)
1882 Oct 7.45	+4.4	−0.9	(−2.5)	(−0.4)	A	Wilson (Cincinnati)
1882 Oct 7.45	−8.2	+1.8	(−3.4)	(+1.2)	C	Wilson (Cincinnati)
1882 Oct 7.46	−7.4	+1.6	(−2.6)	(+1.1)	C	Wilson (Cincinnati)
1882 Oct 8.17	−12.9	+1.8	−1.3	+0.6	E	Krueger (Kiel)
1882 Oct 8.45	−14.8	+1.8	+0.7	−0.5	F	Schaeberle (Ann Arbor)[c,e]
1882 Oct 9.46	−14.8	+1.8	+1.7	−0.8	F	Schaeberle (Ann Arbor)[c,e]
1882 Oct 11.10	−15.5	+2.1	−1.9	+0.3	E	Elkin (Cape of Good Hope)
1882 Oct 13.11	−7.2	+1.5	−0.7	+0.4	C	Elkin (Cape of Good Hope)
1882 Oct 13.11	−13.7	+2.8	+1.3	+0.5	E	Elkin (Cape of Good Hope)
1882 Oct 13.11	−21.1	+4.3	−1.2	+0.4	F	Elkin (Cape of Good Hope)
1882 Oct 14.10	−12.1	+1.5	−0.1	−0.3	D	Finlay (Cape of Good Hope)
1882 Oct 14.40	+9.5	−2.6	−0.4	−1.1	A	Wilson (Cincinnati)
1882 Oct 14.40	−8.1	+2.2	−1.2	+1.0	C	Wilson (Cincinnati)
1882 Oct 14.43	−7.0	+1.4	−0.1	+0.2	C	Schaeberle (Ann Arbor)
1882 Oct 14.46	+6.5	−1.8	(−3.4)	(−0.3)	A	Wilson (Cincinnati)
1882 Oct 14.46	−6.5	+1.8	+0.4	+0.6	C	Wilson (Cincinnati)
1882 Oct 15.28	−6.4	+0.9	+0.7	−0.4	C	Cruls (Rio de Janeiro)
1882 Oct 15.43	−7.2	+1.3	0.0	−0.1	C	Schaeberle (Ann Arbor)
1882 Oct 17.12	+10.7	−2.6	−0.4	−0.6	A	Finlay (Cape of Good Hope)
1882 Oct 18.45	−8.5	+1.8	−0.4	0.0	C	Schaeberle (Ann Arbor)
1882 Oct 19.20	−14.3	+4.7	+0.5	+1.7	D	Büttner (Karlsruhe)[c,f]
1882 Oct 19.35	−18.8	+6.9	(+0.5)	(+2.6)	E	Thome (Córdoba)
1882 Oct 20.43	+8.6	−2.6	(−3.9)	(+0.1)	A	Wilson (Cincinnati)
1882 Oct 20.43	−9.6	+2.9	−0.9	+0.8	C	Wilson (Cincinnati)
1882 Oct 20.44	−10.5	+2.6	−1.8	+0.5	C	Schaeberle (Ann Arbor)
1882 Oct 20.46	−7.9	+2.4	+0.8	+0.3	C	Wilson (Cincinnati)
1882 Oct 21.17	−18.9	+4.6	+1.6	−0.4	E	Cacciatore (Palermo)[c,g]
1882 Oct 21.40	−10.6	+3.1	−1.6	+0.8	C	Tucker (Albany)[c,h]
1882 Oct 21.43	−10.0	+2.9	−1.0	+0.6	C	Schaeberle (Ann Arbor)
1882 Oct 21.44	−7.6	+2.3	+1.4	0.0	C	Wilson (Cincinnati)
1882 Oct 21.45	−9.8	+2.9	−0.8	+0.6	C	Wilson (Cincinnati)
1882 Oct 26.41	−11.9	+4.1	−1.5	+0.8	C	Schaeberle (Ann Arbor)
1882 Oct 30.40	−11.4	+4.9	+0.1	+0.6	C	Wilson (Cincinnati)
1882 Oct 30.42	−14.5	+6.3	(−3.1)	(+2.0)	C	Wilson (Cincinnati)
1882 Oct 31.23	−10.0	+4.7	+1.7	+0.2	C	Common (Ealing)
1882 Nov 3.45	−10.7	+5.1	+1.7	−0.4	C	Wilson (Cincinnati)
1882 Nov 3.45	−22.0	+10.5	0.0	+1.4	D	Wilson (Cincinnati)
1882 Nov 3.46	−11.2	+5.3	+1.2	−0.2	C	Winlock (Washington)[c,d]
1882 Nov 3.46	−15.3	+7.3	(−2.8)	(+1.8)	C	Sampson (Washington)[c,d]
1882 Nov 3.46	−20.5	+9.7	+1.5	+0.7	D	Winlock (Washington)[c,d]
1882 Nov 3.46	−22.8	+10.8	−0.8	+1.8	D	Sampson (Washington)[c,d]
1882 Nov 3.47	−14.4	+6.8	−1.9	+1.3	C	Wilson (Cincinnati)
1882 Nov 4.42	−10.8	+5.0	+1.8	−0.8	C	Wilson (Cincinnati)
1882 Nov 4.42	−23.7	+11.0	−1.3	+1.4	D	Wilson (Cincinnati)
1882 Nov 4.44	−19.1	+8.9	(+3.3)	(−0.7)	D	Wilson (Cincinnati)
1882 Nov 6.46	−15.3	+6.2	(−2.3)	(−0.3)	C	Sampson (Washington)[c,i]
1882 Nov 6.46	−24.7	+10.0	−1.6	−0.7	D	Sampson (Washington)[c,i]
1882 Nov 8.39	−12.9	+8.5	+0.5	+1.4	C	Wilson (Cincinnati)
1882 Nov 8.44	−16.0	+10.6	(−2.6)	(+3.4)	C	Wilson (Cincinnati)
1882 Nov 14.46	−20.1	+11.8	(+5.0)	(−4.2)	D	Winlock (Washington)[c,j]
1882 Nov 16.40	−13.9	+11.5	+0.3	+1.1	C	Winlock (Washington)
1882 Nov 19.40	−15.5	+13.0	−1.3	+1.3	C	Winlock (Washington)
1882 Nov 19.40	−15.5	+13.1	−1.4	+1.4	C	Sampson (Washington)

TABLE 4—*Continued*

Date (UT)	Observed Offset[a] in R.A.[b] (arcsec)	Observed Offset[a] in Decl. (arcsec)	Residual $O-C$ in R.A.[b] (arcsec)	Residual $O-C$ in Decl. (arcsec)	Condensation	Observer and Site
1882 Nov 19.40	−25.3	+21.2	0.0	+1.5	D	Sampson (Washington)
1882 Nov 20.13	−16.8	+11.6	(−2.7)	(−0.5)	C	Palisa (Vienna)
1882 Nov 30.30	−10.1	+18.4	(+2.1)	(+1.4)	C	Bigourdan (Tartenson)
1882 Dec 4.34	−11.7	+22.1	(−1.1)	(+3.1)	C	Bigourdan (Tartenson)
1882 Dec 4.40	−10.3	+20.7	+0.3	+1.7	C	Sampson (Washington)
1882 Dec 4.40	−15.3	+30.8	(+4.1)	(−1.6)	D	Winlock (Washington)
1882 Dec 6.36	−13.4	+26.4	(−3.7)	(+6.5)	C	Sampson (Washington)[c,g]
1882 Dec 25.12	+3.7	+24.7	−0.6	−0.7	C	Thome (Córdoba)[c,g]
1882 Dec 30.20	−10.2	−35.3	+1.0	+0.1	A	Wendell (Cambridge)
1882 Dec 30.91	+6.4	+25.1	(−3.0)	(−0.3)	C	Trépied (Algiers)
1882 Dec 31.88	+5.7	+25.5	(−4.6)	(+0.2)	C	Trépied (Algiers)
1883 Jan 3.91	+6.9	+19.4	(−6.0)	(−5.5)	C	Trépied (Algiers)
1883 Jan 13.22	−32.2	−23.1	(−4.9)	(+8.5)	A	Wendell (Cambridge)
1883 Jan 27.87	+27.5	+15.4	−1.0	−1.7	C	Common (Ealing)
1883 Jan 30.88	+32.6	+19.5	(+3.0)	(+3.6)	C	Bigourdan (Paris)
1883 Feb 2.16	+31.1	+16.1	+0.7	+1.0	C	Winlock (Washington)[c,k]
1883 Feb 2.16	+68.4	+35.4	−0.4	+1.1	E	Winlock (Washington)[c,k]
1883 Feb 2.95	+30.9	+13.2	+0.2	−1.5	C	Bigourdan (Paris)
1883 Feb 5.82	+31.6	+12.2	+0.2	−1.5	C	Bigourdan (Paris)
1883 Feb 5.83	+34.9	+15.3	(+3.5)	(+1.6)	C	Schur (Strasbourg)
1883 Feb 6.18	+34.0	+10.7	(+2.5)	(−2.9)	C	Wendell (Cambridge)
1883 Feb 6.86	+32.2	+12.2	+0.5	−1.1	C	Bigourdan (Paris)
1883 Feb 8.20	+36.0	+16.4	(+4.0)	(+3.7)	C	Wendell (Cambridge)
1883 Feb 9.87	+32.8	+10.3	+0.4	−1.9	C	Bigourdan (Paris)
1883 Feb 11.90	+34.6	+14.4	(+1.9)	(+3.0)	C	Baillaud (Toulouse)[c,l]
1883 Feb 11.92	+32.3	+13.5	(−0.4)	(+2.0)	C	Schur (Strasbourg)
1883 Feb 13.82	+32.6	+11.1	−0.5	+0.3	C	Bigourdan (Paris)
1883 Feb 24.83	+32.3	+7.6	−1.7	+0.3	C	Common (Ealing)
1883 Feb 27.03	−46.0	−10.1	+0.9	−0.9	A	Hall (Washington)
1883 Feb 27.03	+33.7	+7.4	−0.3	+0.8	C	Hall (Washington)
1883 Feb 28.05	−48.1	−9.2	−1.2	−0.4	A	Hall (Washington)
1883 Feb 28.05	+34.0	+6.5	−0.1	+0.2	C	Hall (Washington)
1883 Feb 28.06	+55.8	+10.7	−1.9	−0.1	D	Hall (Washington)
1883 Mar 2.82	+34.7	+6.6	+0.7	+1.0	C	Bigourdan (Paris)
1883 Mar 3.81	+33.2	+5.9	−0.8	+0.6	C	Bigourdan (Paris)
1883 Mar 8.05	+29.9	+4.0	(−3.9)	(−0.3)	C	Wendell (Cambridge)
1883 Mar 8.05	+52.6	+4.4	(−4.9)	(−2.8)	D	Wendell (Cambridge)
1883 Mar 14.01	+25.6	+3.3	(−7.8)	(+0.4)	C	Wendell (Cambridge)
1883 Apr 3.04	+34.9	+2.8	(+3.9)	(+3.5)	C	Wendell (Cambridge)
1883 Apr 7.03	+36.3	−1.9	(+5.9)	(−0.6)	C	Wendell (Cambridge)

[a] Referred to position of condensation B; parenthesized residuals indicate rejected offsets.
[b] Including factor cos (decl.).
[c] Observer reported separation distance only.
[d] Reduced with help of position angle measured at nearly the same time by Wilson (Cincinnati).
[e] Reduced with help of position angle averaged from three measurements made at nearly the same time by Krueger (Kiel) and Erck (Bray).
[f] Reduced with help of position angle averaged from two measurements made at nearly the same time by Kortazzi (Nikolayev) and Thome (Córdoba).
[g] Reduced with help of position angle measured at nearly the same time by Elkin (Cape of Good Hope).
[h] Reduced with help of position angle averaged from two measurements made at nearly the same time by Schaeberle (Ann Arbor) and Wilson (Cincinnati).
[i] Reduced with help of position angle averaged from three measurements made at nearly the same time by Elkin (Cape of Good Hope) and Kortazzi (Nikolayev).
[j] Reduced with help of position angle averaged from two measurements made at nearly the same time by Elkin (Cape of Good Hope).
[k] Reduced with help of position angle averaged from two measurements made at nearly the same time by Elkin (Cape of Good Hope) and Bigourdan (Paris).
[l] Reduced with help of position angle measured at nearly the same time by Schur (Strasbourg).

Table 4 shows our fitting of a total of 102 offsets of condensations A, C, D, E, and F from condensation B of C/1882 R1, which for the pair B and C span more than 6 months. It was necessary to reorganize and overhaul Kreutz's sets of separation distances and position angles, as well as to sort out and correct several misidentifications of the observed condensations. Requiring that the positional residuals from each fragmentation solution not exceed $\pm 2''$ in both right ascension and declination, we rejected 33 data points. In a second solution for C relative to B, based on a tighter limit of $\pm 1.5''$, 11 additional data points were rejected.

Table 5 presents a total of 41 offsets between the two widely observed condensations of C/1965 S1. These data, reported by

TABLE 5

Astrometric Offsets between Nucleus Condensations A and B of Comet C/1965 S1 (Ikeya-Seki) (Equinox B1950.0)

Date (UT)	Observed Offset[a] in R.A.[b] (arcsec)	Observed Offset[a] in Decl. (arcsec)	Residual O − C from Solution 1 in R.A.[b] (arcsec)	Residual O − C from Solution 1 in Decl. (arcsec)	Residual O − C from Solution 2 in R.A.[b] (arcsec)	Residual O − C from Solution 2 in Decl. (arcsec)	Observer and Site
1965 Nov 4.53	−13.5	+3.6					Pohn (Flagstaff)[c,d]
1965 Nov 5.11	−8.7	+2.3	−1.0[e]	+0.7[e]	0.0[e]	+0.7[e]	Lourens (Cape of Good Hope)[d]
1965 Nov 5.53	−15.5	+4.1	−1.1[f]	+0.6[f]	−0.1[f]	+0.6[f]	Pohn (Flagstaff)[c,d]
1965 Nov 6.10	−14.5	+3.9					Andrews (Bloemfontein)[c,d]
1965 Nov 6.84	−9.6	+2.8	−0.3[g]	+0.4[g]	+0.9[g]	+0.4[g]	Ikeya (Shizuoka)[c,d]
1965 Nov 10.10	−17.7	+3.1	(−2.8)	(−1.2)	−1.3	−1.3	Thackeray (Pretoria)
1965 Nov 12.10	−17.7	+3.1	(−1.0)	(−2.1)	(+0.6)	(−2.3)	Thackeray (Pretoria)
1965 Nov 12.83	−17.3	+4.9	0.0	−0.7	(+1.6)	(−0.9)	Tomita (Tokyo)
1965 Nov 12.83	−19.4	+6.3	(−2.1)	(+0.7)	−0.4	+0.5	Tomita (Tokyo)
1965 Nov 12.84	−16.8	+7.8	(+0.5)	(+2.2)	(+2.2)	(+2.0)	Tomita (Tokyo)
1965 Nov 14.10	−15.7	+8.5	(+2.6)	(+2.2)	(+4,4)	(+1.9)	Lourens (Cape of Good Hope)
1965 Nov 17.31	−21.0	+9.2	−0.1	+0.8	(+1.9)	(+0.5)	Pereyra & Rodríguez (Córdoba)
1965 Nov 17.31	−21.0	+9.3	−0.1	+0.9	(+1.9)	(+0.6)	Pereyra & Rodríguez (Córdoba)
1965 Nov 17.32	−22.8	+7.7	(−1.9)	(−0.7)	+0.2	−1.1	Pereyra & Rodríguez (Córdpba)
1965 Nov 19.29	−25.4	+9.7	(−3.1)	(−0.1)	−0.9	−0.6	Pereyra & Rodríguez (Córdoba)
1965 Nov 19.30	−25.6	+11.1	(−3.2)	(+1.3)	−1.0	+0.8	Pereyra & Rodríguez (Córdoba)
1965 Nov 19.31	−24.0	+7.7	(−1.6)	(−2.1)	(+0.6)	(−2.6)	Pereyra & Rodríguez (Córdoba)
1965 Nov 21.82	−23.4	+13.6	(+0.6)	(+1.7)	(+3.0)	(+1.1)	Tomita (Tokyo)
1965 Nov 22.79	−24.6	+13.2	0.0	+0.4	(+2.4)	(−0.3)	Tomita (Tokyo)
1965 Nov 22.80	−24.7	+15.9	(−0.1)	(+3.1)	(+2.3)	(+2.4)	Tomita (Tokyo)
1965 Nov 22.85	−25.2	+14.5	(−0.6)	(+1.7)	(+1.8)	(+1.0)	Tomita (Tokyo)
1965 Nov 23.08	−22.1	+10.9	(+2.6)	(−2.2)	(+5.1)	(−2.9)	Lourens (Cape of Good Hope)
1965 Nov 24.05	−25.9	+14.2	−0.7	+0.2	(+1.8)	(−0.6)	Thackeray et al. (Pretoria)
1965 Nov 24.08	−25.2	+14.2	+0.1	+0.2	(+2.6)	(−0.6)	Thackeray et al. (Pretoria)
1965 Nov 27.32	−31.6	+22.0	(−5.0)	(+4.6)	(−2.3)	(+3.6)	Pereyra & Rodríguez (Córdoba)
1965 Nov 27.33	−31.1	+20.4	(−4.5)	(+3.0)	(−1.8)	(+2.0)	Pereyra & Rodríguez (Córdoba)
1965 Nov 27.79	−28.2	+19.2	−1.4	+1.3	+1.3	+0.2	Tomita (Tokyo)
1965 Nov 30.08	−25.7	+19.2	+1.4	−1.3	(+4.3)	(−2.6)	Lourens (Cape of Good Hope)
1965 Dec 1.56	−26.9	+21.8	+0.4	−0.5	(+3.2)	(−2.0)	Tammann (Palomar Mountain)
1965 Dec 2.56	−28.6	+23.3	−1.3	−0.3	(+1.5)	(−1.9)	Tammann (Palomar Mountain)
1965 Dec 3.24	−28.2	+27.5	(−1.1)	(+3.0)	(+1.8)	(+1.4)	Pereyra & Rodríguez (Córdoba)
1965 Dec 3.26	−29.9	+26.8	(−2.8)	(+2.3)	+0.1	+0.7	Pereyra & Rodríguez (Córdoba)
1965 Dec 7.30	−24.9	+28.4	+0.9	−1.4	(+3.7)	(−3.5)	Pereyra & Rodríguez (Córdoba)
1965 Dec 22.99	−5.6	+46.7	+0.1	−0.2	(+1.5)	(−3.9)	Thackeray et al.. (Pretoria)
1965 Dec 23.93	−4.1	+47.8	−0.2	+0.4	(+1.1)	(−3.4)	Thackeray et al.. (Pretoria)
1965 Dec 23.94	−3.2	+46.7	+0.7	−0.7	(+2.0)	(−4.5)	Thackeray et al.. (Pretoria)
1965 Dec 24.17	−3.3	+47.6	+0.2	0.0	(+1.4)	(−3.7)	Pereyra & Rodríguez (Córdoba)
1965 Dec 24.20	−3.2	+48.0	+0.2	+0.4	(+1.4)	(−3.3)	Pereyra & Rodríguez (Córdoba)
1965 Dec 24.22	−2.7	+47.8	+0.7	+0.2	(+1.8)	(−3.6)	Pereyra & Rodríguez (Córdoba)
1965 Dec 31.38	+9.4	+49.2	−1.4	+0.2	(−1.5)	(−3.8)	Tammann (Palomar Mountain)
1966 Jan 14.33	+36.9	+44.8	(+1.9)	(+2.8)	−0.1	−0.7	Tammann (Palomar Mountain)

[a] In the sense of condensation B minus condensation A; parenthesized residuals indicate rejected offsets.
[b] Including factor cos (decl.).
[c] Observer reported separation distance only; offsets reduced with help of position angle derived from formula by Lourens (1966).
[d] Residuals from averaged offsets by the two observers.
[e] Residual is an average of 1965 Nov 4.53 and Nov 5.11.
[f] Residual is an average of 1965 Nov 5.11 and Nov 5.53.
[g] Residual is an average of 1965 Nov 6.10 and Nov 6.84.

Andrews (1965), Hirose (1965), Iannini (1966), Lourens (1966), Marsden (1967), Pohn (1965), Tammann (1966), Thackeray (1965), and Tomita (1965a, 1965b), cover a period of more than 2 months. As with C/1882 R1, it was not possible to solve simultaneously for the fragmentation time and the separation velocity, so we again used the two-parameter model. A requirement for positional residuals to be confined to $\pm 1.5''$ called for 11 data points to be rejected. The remaining offsets were accommodated by one of two alternative solutions that we present. The data from unconfirmed reports of a possible third nucleus condensation (Pohn 1965) and inconsistent observations by Milet (1965a, 1965b) are not included in Table 5.

Table 6 summarizes our results for the time of tidally driven fragmentation for comets C/1882 R1 and C/1965 S1. Also listed are the data from two solutions for the on-train fragments of comet D/Shoemaker-Levy 9 (Sekanina et al. 1998). Strikingly, all fragmentation occurred *after* the time of closest approach to the perturbing body, providing evidence against a strengthless agglomerate (or a rubble pile) comet model dominated by effects of gravity. Instead, the comets appear to be poorly cemented, yet

TABLE 6

Timing of Tidal Fragmentation for Nuclei of Comets C/1882 R1, C/1965 S1, and D/1993 F2

Comet	Nuclear Fragments Involved	Separation Time[a] (hr)	Number of Data Points	rms Residual	Comment
C/1882 R1[b]	B, A	2.7 ± 1.0	7	±0.77	
	B, C	1.4 ± 0.4	43	±1.00	
	B, C	1.9 ± 0.4	32	±0.85	Residuals ±1.5″ or greater rejected
	B, D	2.6 ± 0.9	9	±1.24	
	B, E	1.8 ± 0.9	5	±1.20	
	B, F	0.5 ± 0.6	5	±1.02	
C/1965 S1[c]	A, B	0.5 ± 0.1	21	±0.76	Solution 1
	A, B	0.9 ± 0.2	11	±0.78	Solution 2
D/1993 F2	E–W	3.1 ± 0.2[d]	144	±0.11[e]	Eight fragments: E, G, H, K, L, Q, S, W
	A–W	2.3 ± 0.7[d]	41	±0.18[e]	11 fragments: A, C, D, E–W

[a] After perihelion (C/1882 R1, C/1965 S1) or after perijove (D/1993 F2).
[b] Offsets yielding residuals ±2.0″ or greater were rejected, unless indicated otherwise in comments.
[c] Offsets yielding residuals ±1.5″ or greater were rejected.
[d] Initiation time of tidal fragmentation, as derived by Sekanina et al. (1998) from nuclear train orientation.
[e] Mean scatter among fragments' averaged deviations from their train.

cohesive, objects, as recently also indicated by the close-up imaging results for 81P/Wild (Brownlee et al. 2004) and 9P/Tempel (A'Hearn 2006). The nominal range of fragmentation times in Table 6, 0.5–3.1 hr, is crudely consistent with an independently derived time of disintegration of C/1887 B1 (Sekanina 1984), 5.8 ± 0.8 hr after perihelion. Except for its tidal nature, this episode may have been similar to that experienced by C/1999 S4 (Weaver et al. 2001). This kind of phenomenon was also exhibited by several companion nuclei of comet 73P/Schwassmann-Wachmann, including B and G, during 2006 April–May (e.g., Weaver et al. 2006).

The range of separation times for the tidally driven fragmentation events of the three split comets in Table 6 is centered on 1.8 hr or 0.075 days after closest approach. A median and an average of the 10 entries are also very close to this time. We adopt this value as our best estimate for the tidal fragmentation time of X/1106 C1.

5. RANGE OF SEPARATION VELOCITIES

On this assumption, we show in Table 7 for C/1882 R1 and in Table 8 for C/1965 S1 that from three-parameter solutions the separation velocities relative to the center of mass never exceed 4 m s^{-1}, the radial and transverse components are always of the same sign, and the poorly determined normal component is very small and has a tendency to be of the opposite sign. As extreme approximations, the radial and transverse directions constrain the separation velocity vector to not more than 5 m s^{-1} when referred to the center of mass. To account for possible additional fainter (undetected) fragments slightly farther from the center of mass, we accept an absolute upper limit of $(V_R)_{\rm lim} = \pm 5.5$ m s^{-1} on the radial component.

We find our results to be in general agreement with those derived by Sekanina et al. (1998) for D/Shoemaker-Levy 9, whose nucleus condensations separated with the relative velocities of up to 1.7 m s^{-1}. Because of strong evidence for their rotational nature, we expect that the separation velocities of comets with initially larger nuclei should generally be higher. Sekanina (2002a) estimated the nucleus of C/1882 R1 at 50 km across, while the nucleus of D/Shoemaker-Levy 9 was found to be about a factor of 5 smaller (Sekanina et al. 1998).

6. TEMPORAL EXTENT OF THE KREUTZ SYSTEM

An excellent parameter constraining fragmentation scenarios for the Kreutz system is the temporal extent of its members' arrival times, restricted by the narrow range of separation velocities that protofragments can acquire on the tidally driven breakup of their parent. In addition, this temporal extent determines the orbital location of the system's center of mass at a given stage of evolution.

6.1. *The Two-Superfragment Model of Paper I*

Developing the two-superfragment model in Paper I, we assumed that no Kreutz comet arrived at perihelion in between 1106 and 1843. Since the second brightest sungrazer known, C/1843 D1, preceded C/1882 R1 by nearly 40 yr, while the members of the 20th century cluster arrived ~80 yr after C/1882 R1, the system's center of mass could reasonably be assumed to be positioned at (or close to) C/1882 R1. Two unsettled questions concern implications of possible additional clusters of related sungrazers: (1) those that had arrived during the period of some 300 yr or so before 1843, as proposed by Kreutz (1901), Marsden (1967, 1989), Hasegawa & Nakano (2001), and Strom (2002), and (2) those that may arrive in the future, in the course of the 21st century and perhaps beyond. The existence of any such clusters would dramatically alter the orbital position and long-term motion of the center of mass of the Kreutz system and would require *new scenarios* for the system's origin and evolution.

6.2. *Search for Additional Bright Sungrazers*

To assemble as much information on bright sungrazers as possible, we have searched several sources, especially compilations of historical records of comets that address the subject of the Kreutz system and focus primarily on the pre-1843 period of time. General compilations with no reference to comets' sungrazer nature were deemed unhelpful and were not consulted.

One of the most relevant sources, Kreutz (1901), offers, besides the four major objects between 1843 and 1887, detailed accounts of seven comets observed between the second half of the 17th century and the end of the 19th century. Of these, only comets C/1668 E1, X/1702 D1, and X/1882 K1 "Tewfik" (seen during the total solar eclipse of 1882 May 17 UT) were considered by Kreutz as potential sungrazers. The perihelion distance of 0.0666 AU in one of his orbital sets for C/1668 E1, which is listed in the comet orbit catalog (Marsden & Williams 2003), is 1 order of magnitude greater than expected for the sungrazers and is somewhat misleading; in the same paper Kreutz (on p. 74) concluded that C/1668 E1 moved in the orbital plane of

TABLE 7

Relative Separation Velocities for Nucleus Condensations of Comet C/1882 R1 from Astrometric Offsets

Relative to	Fragment	Three-Parameter Solution: V_{sep} (m s^{-1})	V_R (m s^{-1})	V_T (m s^{-1})	V_N (m s^{-1})	rms Residual (arcsec)	Case $V_{sep} = \|V_R\|$: V_{sep} (m s^{-1})	rms Residual (arcsec)	Case $V_{sep} = \|V_T\|$: V_{sep} (m s^{-1})	rms Residual (arcsec)	Number of Data Points
B	A	1.89 ± 0.30	−1.33 ± 0.27	−1.15 ± 0.29	+0.69 ± 0.39	±0.76	2.40 ± 0.04	±1.24	2.60 ± 0.04	±1.28	7
	C	1.34 ± 0.12	+1.14 ± 0.11	+0.64 ± 0.12	−0.31 ± 0.20	±0.97	1.75 ± 0.01	±1.13	1.85 ± 0.02	±1.43	43
	D	2.57 ± 0.34	+1.19 ± 0.26	+1.84 ± 0.26	−1.35 ± 0.50	±1.00	3.08 ± 0.07	±2.00	3.06 ± 0.06	±1.76	9
	E	3.03 ± 0.56	+2.49 ± 0.53	+1.55 ± 0.56	−0.78 ± 0.82	±1.21	3.96 ± 0.08	±1.63	4.22 ± 0.11	±2.21	5
	F[a]	4.0 ± 1.1	+3.0 ± 1.3	+2.2 ± 1.1	(−1.5)	±1.01	5.76 ± 0.17	±0.99	4.78 ± 0.22	±1.59	5
Center of mass[b]	A	2.52 $\mathcal{B}$	−1.88	−1.46	+0.84	...	3.24 $\mathcal{B}$	...	3.49 $\mathcal{B}$	...	...
	B	0.65 $\mathcal{B}$	−0.55	−0.31	+0.15	...	0.84 $\mathcal{B}$	...	0.89 $\mathcal{B}$	...	...
	C	0.69 $\mathcal{F}$	+0.59	+0.33	−0.16	...	0.91 $\mathcal{F}$	...	0.96 $\mathcal{F}$	...	...
	D	2.05 $\mathcal{F}$	+0.64	+1.53	−1.20	...	2.24 $\mathcal{F}$	...	2.17 $\mathcal{F}$	...	...
	E	2.39 $\mathcal{F}$	+1.94	+1.24	−0.63	...	3.12 $\mathcal{F}$	...	3.33 $\mathcal{F}$	...	...
	F[a]	3.4 $\mathcal{F}$	+2.5	+1.9	(−1.3)	...	4.92 $\mathcal{F}$	...	3.89 $\mathcal{F}$	...	...

[a] This is a two-parameter solution in which the forced value of V_N is parenthesized; results from the three-parameter solution are indeterminate, with formal errors greatly exceeding the derived values. Forcing instead $V_N = -1.0$ m s^{-1}, we find $V_{sep} = 4.2 \pm 1.2$ m s^{-1}, $V_R = +3.7 \pm 1.3$ m s^{-1}, and $V_T = +1.7 \pm 1.1$ m s^{-1}, with the rms residual of $\pm 1.02''$.

[b] In columns for V_{sep}, $\mathcal{B}$ indicates that relative to the motion of the center of mass the separation velocity points backward, in a general direction opposite the comet's orbital motion, while $\mathcal{F}$ indicates that the separation velocity points forward.

TABLE 8

Relative Separation Velocities for Nucleus Condensations of Comet C/1965 S1 from Astrometric Offsets

Relative to	Fragment	Three-Parameter Solution: V_{sep} (m s^{-1})	V_R (m s^{-1})	V_T (m s^{-1})	V_N (m s^{-1})	rms Residual (arcsec)	Case $V_{sep} = \|V_R\|$: V_{sep} (m s^{-1})	rms Residual (arcsec)	Case $V_{sep} = \|V_T\|$: V_{sep} (m s^{-1})	rms Residual (arcsec)	Number of Data Points
A	B[a]	2.56 ± 0.11	+2.52 ± 0.10	+0.37 ± 0.09	−0.29 ± 0.40	±0.75	2.90 ± 0.02	±0.87	2.90 ± 0.06	±3.38	21
	B[b]	2.54 ± 0.20	+2.43 ± 0.20	+0.73 ± 0.20	+0.04 ± 0.47	±0.78	3.14 ± 0.03	±1.00	3.14 ± 0.08	±2.38	11
Center of mass[c]	A[a]	0.16 $\mathcal{B}$	−0.16	−0.02	+0.02	...	0.18 $\mathcal{B}$	...	0.18 $\mathcal{B}$	...	...
	A[b]	0.16 $\mathcal{B}$	−0.15	−0.05	0.00	...	0.20 $\mathcal{B}$	...	0.20 $\mathcal{B}$	...	...
	B[a]	2.40 $\mathcal{F}$	+2.36	+0.35	−0.27	...	2.72 $\mathcal{F}$	...	2.72 $\mathcal{F}$	...	...
	B[b]	2.38 $\mathcal{F}$	+2.28	+0.68	+0.04	...	2.94 $\mathcal{F}$	...	2.94 $\mathcal{F}$	...	...

[a] Solution 1.

[b] Solution 2.

[c] In columns for V_{sep}, $\mathcal{B}$ indicates that relative to the motion of the center of mass the separation velocity points backward, in a general direction opposite the comet's orbital motion, while $\mathcal{F}$ indicates that the separation velocity points forward.

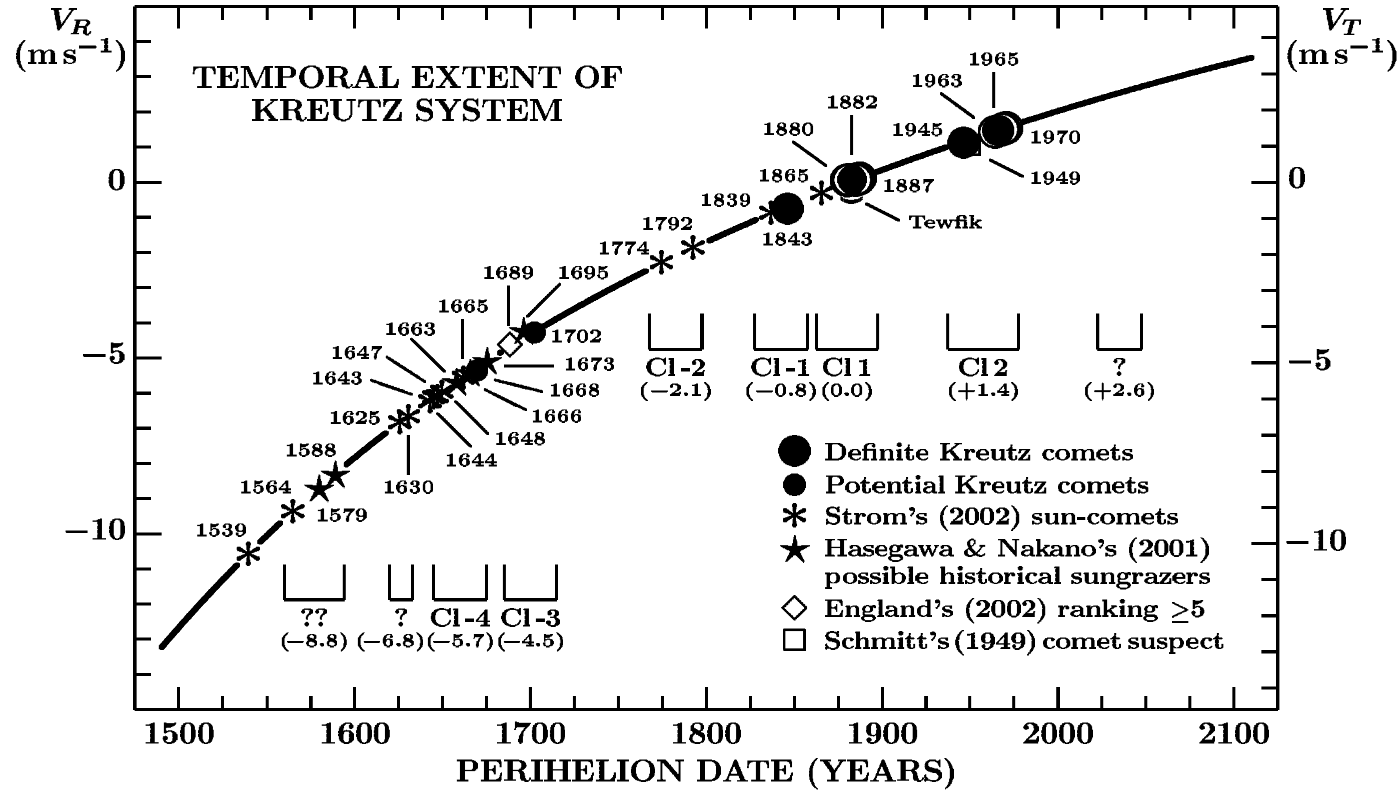

FIG. 4.—Temporal extent and clustering of the Kreutz system. The various symbols used for the sungrazers are explained in the figure. The separation velocity (radial, V_R, on the left; transverse, V_T, on the right) needed to explain the time gap between a comet and C/1882 R1 (for which $V_R = V_T = 0$; the two-superfragment model from Paper I) is plotted as a function of the perihelion date. The solid line displays eq. (8). Also shown are the various sungrazer clusters (Cl 2, Cl 1, etc.) and their representative V_R (in parentheses). Values of $|V_R| \geq 5.5$ m s^{-1} are considered excessive and unrealistic.

C/1843 D1 (rather than C/1882 R1) and its astrometric observations did not contradict the elements of C/1843 D1 for an appropriate perihelion time. An observation of comet X/1882 K1, unknown to Kreutz (1901), shows it to move in an orbit also consistent with that of C/1843 D1 (Marsden 1967).

Our next source is a list of daytime observations of bright objects near the Sun recently compiled by Strom (2002) from Chinese annals. He argues that comets, sungrazers in particular, are the best candidates to account for these events. Calling them "sun-comets," Strom points out that during the last two centuries ~60% of comets seen with the naked eye near the Sun in broad daylight have been sungrazers. All but one of the 14 objects listed by Strom were recorded between 1539 and 1865, seven appeared during a period of 40 yr centered on 1645, and four between 1643 and 1648. At least some of these objects must have been sungrazers. We also notice a fair coincidence with C/1668 E1, which was not a sun-comet.

A further argument in favor of a cluster of sungrazers in the 17th century is provided by Hasegawa & Nakano's (2001) compilation of 24 possible Kreutz comets starting at 5 BCE. Based on a number of historical records with positional and temporal information, their list contains eight candidates between 1579 and 1702, including four in 1663–1673. Five of them are new, while the remaining three, C/1668 E1, C/1695 U1, and X/1702 D1, were already checked by Kreutz (1901). A disagreement exists only for C/1695 U1, which was ruled out as a member of this system by Kreutz but considered possible by Hasegawa & Nakano (2001).

A less restrictive list of candidate sungrazers was recently published by England (2002). Developing a set of nine criteria, he ranked comets on a scale from 0 (not a sungrazer) to 10 (a definite sungrazer). Among his 62 assembled objects between 1375 BCE and AD 1702, none were ranked 0 or 10, 26 were ranked 1 or 2, and 11 were ranked 5–9. Only four of these 11 comets arrived in the 16th century or more recently, all of them already scrutinized by Kreutz (1901). Unlike Kreutz, England considered both C/1689 X1 and C/1695 U1 as probable members of the system, ranking them 9; in the case of C/1689 X1, England also disagreed with Hasegawa & Nakano (2001), who regarded this object an unlikely candidate.

The last of our sources of information is a short report by Schmitt (1949), referring to a rectilinear, 70° long and 1° wide feature observed in the evening of 1949 July 16 at the Algiers Observatory. The feature extended from the horizon in the northwest to low elevations in the north-northeast and disappeared below the horizon completely about 1 hr after it was first detected. Schmitt described its appearance as resembling a reproduction of the 1843 comet. Although he offered several possible explanations for the phenomenon (including aurora borealis), a bright tail of a comet near the Sun (which was more than 8° below the horizon in the northwest) appears to be the most likely one. A preliminary analysis of the report suggests that, if belonging to a sungrazer, the phenomenon could be a surviving tail of an object that catastrophically disintegrated very shortly before reaching perihelion a few days earlier. The feature was detected neither before July 16 (perhaps because the tail's length was then much shorter) nor afterward (possibly because it became too faint). A preperihelion disappearance is a common trait of the *SOHO* sungrazers (e.g., Biesecker et al. 2002; Sekanina 2003), but the sungrazer of 1949 must have been much brighter. The only known case of a persistently surviving tail of a headless sungrazer is of course comet C/1887 B1 (Sekanina 1984), which was found to have disintegrated a fraction of a day after perihelion (§ 4).

TABLE 9
List of Definite and Possible Bright Kreutz Sungrazers Observed during the 16th–20th Centuries and the Proposed Clusters

Proposed Cluster	Object	Category[a]	Representative Radial Velocity V_R (m s^{-1})		
			Paper I	Scenario A	Scenario B
	1539	St	...	...	...
1564–1588	1564	St	(−8.8)	(−6.2)	−5.3
	1579	HN	...	...	...
	1588	HN	...	...	...
1625–1630	1625	St	(−6.8)	−4.5	−3.6
	1630	St	...	...	...
1643–1673	1643	St	(−5.7)	−3.7	−2.7
	1644	St	...	...	...
	1647	St	...	...	...
	1648	St	...	...	...
	1663	HN	...	...	...
	1665	St	...	...	...
	1666	HN	...	...	...
	C/1668 E1	Kp, HN, En	...	...	...
	1673	HN	...	...	...
1689–1702	C/1689 X1	En[b,c]	−4.5	−2.6	−1.7
	C/1695 U1	HN, En[b]	...	...	...
	X/1702 D1	Kp, HN, En	...	...	...
1774–1792	1774	St	−2.1	−0.6	+0.4
	1792	St	...	...	...
1839–1843	1839	St	−0.8	+0.6	+1.5
	C/1843 D1	Kd	...	...	...
1865–1887	1865	St	0.0	+1.3	+2.2
	C/1880 C1	Kd	...	...	...
	X/1882 K1	Kp	...	...	...
	C/1882 R1	Kd	...	...	...
	C/1887 B1	Kd	...	...	...
1945–1970	C/1945 X1	Kd	+1.4	+2.4	+3.4
	1949	Sc	...	...	...
	C/1963 R1	Kd	...	...	...
	C/1965 S1	Kd	...	...	...
	C/1970 K1	Kd	...	...	...

[a] Kd: definite bright Kreutz sungrazer; Kp: bright comet considered by Kreutz (1901) to be a potential or probable sungrazer; St = sun-comet, a daytime object near the Sun, listed by Strom (2002); HN: possible historical sungrazer, listed by Hasegawa & Nakano (2001); En: historical comet, having England's (2002) sungrazer ranking $\geq$5; Sc: object suspected by Schmitt (1949) to be a bright sungrazer's tail.
[b] This comet was considered by Kreutz (1901) not to be a sungrazer.
[c] This comet is not on the Hasegawa & Nakano (2001) list of possible members of the Kreutz system.

The nonrandom temporal distribution of bright comets of the Kreutz system between 1539 and 1970 is presented in Figure 4. The arrival times are plotted against velocity V_R in the radial direction that a protofragment separating from the parent X/1106 C1 requires at the fragmentation time, $t_{\rm frg} = 1.8$ hr after perihelion, in order to reach its next perihelion T at the given time. The 32 objects, listed in the second column of Table 9, include the eight definite Kreutz comets and 24 probable and potential ones. The relationship $V_R(t_{\rm frg},\, T)$ is not linear, as higher velocities are required to move perihelion by the same amount back in time as forward. Normalized arbitrarily to the arrival time of C/1882 R1, an excellent approximation to the curve displayed in Figure 4 (with the velocity in m s^{-1} and the perihelion time in years) is given by

$$V_R(t_{\rm frg},\, T) = 1.877\tau - 0.1817\tau^2 + 0.04933\tau^3, \qquad (8)$$

where $\tau = (T - 1882.71)/100$ and 1882.71 is the perihelion time of C/1882 R1. For any T, the equivalent value of the transverse separation velocity is $V_T(t_{\rm frg}) = 0.97V_R(t_{\rm frg})$, whereas the equivalent separation velocity along the orbital velocity vector is $V^*_{\rm sep}(t_{\rm frg}) = 0.70V_R(t_{\rm frg})$.

All except the earliest one of the 32 sungrazers are in Table 9 divided into eight chosen clusters. Their boundaries are admittedly somewhat arbitrary, the only applied criterion being that a cluster not span more than 30 yr. It is certainly possible that the cluster of 1643–1673 consists of two subclusters, 1643–1648 and 1663–1673, but the cumulative evidence for a major concentration of potential sungrazers in the middle and late 17th century is in any case very compelling.

The three pre-1680 clusters cannot be accommodated as products of a common parent sungrazer with its center of mass near C/1882 R1 because their required radial components of the separation velocity (fourth column of Table 9) exceed the limit of 5.5 m s^{-1} from § 5. Either these clusters derived from an earlier, independent sequence of fragmentation events (on which we have no other specific information), or, perhaps more probably, the Kreutz system's center of mass reached perihelion long before C/1882 R1.

There is of course no reason why the center of mass of the disintegrating parent comet should coincide with the position of any major fragment. After all, following the breakup of C/1882 R1 this comet's center of mass was found to have been situated between fragments B and C (Sekanina & Chodas 2002a). However, if the arrival time corresponding to the parent comet's center of mass is unknown, fragmentation scenarios for X/1106 C1 become unconstrained, unless this comet can be identified at its previous return to the Sun with an earlier historical comet. Two scenarios based on such presumed identifications are proposed in § 7.

6.3. *Orbital Evidence from the Bright Sungrazers*

Before embarking on this project, we search for information that will help us constrain the perihelion distance and spatial orientation of the protofragments' orbits. The critical issue for developing a specific fragmentation scenario (§ 2.1) is to examine whether the orbit of X/1106 C1 belonged to subgroup II (as assumed in Paper I) or to subgroup I. In practice, we strive to find out which of the two comets, C/1843 D1 (subgroup I) or C/1882 R1 (subgroup II), is orbitally more representative of X/1106 C1. This, incidentally, is the same decision that Kreutz (1901) had to make when he attempted to constrain the orbits of the various suspected members of the system. We consider it unlikely that X/1106 C1 was in an orbit that did not essentially coincide with either subgroup.

Since differential momenta acquired by most fragments in tidally driven events (in the immediate proximity of the Sun) are negligibly small in the elements ω, Ω, i, and q (§ 3), the limited evidence from the definite Kreutz members (§ 2.1) tends to prefer the subgroup I orbital type for X/1106 C1. Consistent with this conclusion are the remarks in § 6.2 that available information on C/1668 E1 and X/1882 K1 also prefers C/1843 D1 to C/1882 R1 for these two sungrazers.

6.4. *Orbital Evidence from the SOHO Sungrazers*

As high-generation fragments that must have experienced a number of nontidal breakup episodes during one revolution about the Sun (Sekanina 2002a, 2002b), the *SOHO* sungrazers should display an orbital distribution determined by their birth signature and broadened by effects of fragmentation events occurring predominantly at large heliocentric distances. In Figure 5 we plot the argument of perihelion ω, the longitude of the ascending node Ω, the inclination i, and the perihelion distance q against the perihelion time for 1000 *SOHO* sungrazers detected on their arrival at the Sun between the beginning of 1996 and 2006 August 9. These plots are based on the sets of parabolic orbital elements derived by B. G. Marsden.[1] The orbits of the *SOHO* sungrazers are more accurate than the orbits of the *SMM* and especially Solwind sungrazers, but they certainly are not as well determined as the orbits of the Kreutz system members observed rather extensively from the ground. The perihelion distance of the *SOHO* minicomets is likely to be the least well determined element, whose value had to be on some occasions forced (B. G. Marsden 2001, private communication; also see Sekanina 2002a; Marsden 2005), but the fixed orientation of the line of apsides was not used to constrain the orbital solutions.

Previous findings suggesting that most of the *SOHO* sungrazers move in orbits close to that of C/1843 D1 (e.g., Sekanina 2002a) are strongly supported by the histograms for the 1000 sungrazers shown in Figure 6. Although the peaks of the distributions do not agree perfectly with C/1843 D1 in each of the four discriminating elements, the correspondence is substantially better than with C/1882 R1. We find no prominent secondary peaks coinciding with the latter comet. A correspondence between the

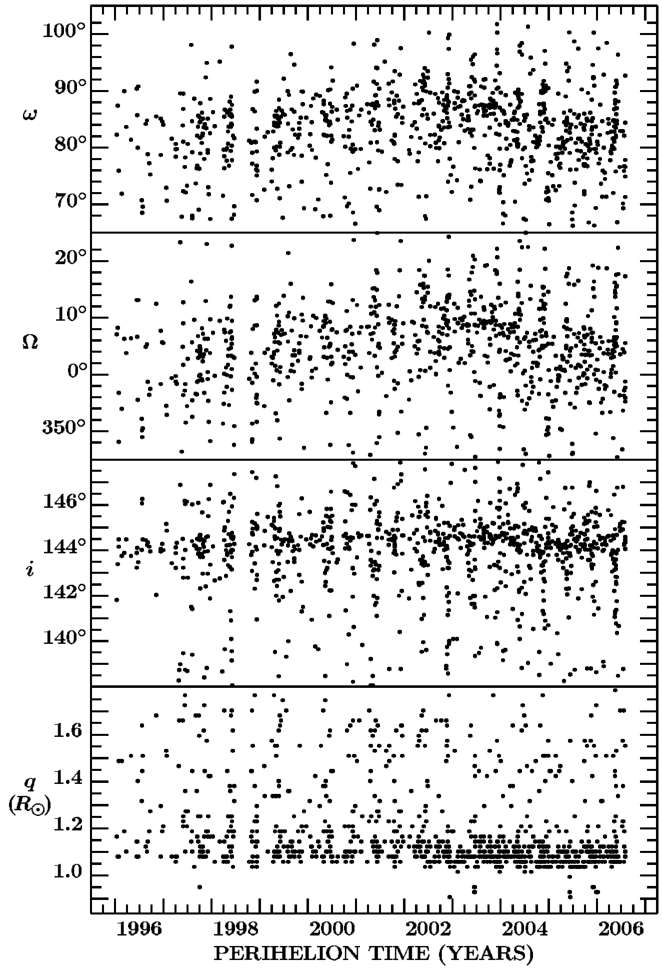

FIG. 5.—Orbital distribution of 1000 *SOHO* sungrazers that arrived at the Sun between the beginning of 1996 and 2006 August 9. The argument of perihelion, the longitude of the ascending node, the inclination, and the perihelion distance, as calculated by Marsden, are plotted against the perihelion time. Systematic temporal variations are apparent especially in the first two plots.

temporal distributions of the elements ω, Ω, i, and q from Figure 5 and the predicted, C/1843 D1–based orbital variations, which is discussed in § 7, also strongly supports the notion that the orbits of C/1843 D1 and the parent comet of the population of *SOHO* sungrazers are very much alike.

Because nontidal breakup events at large heliocentric distances have only a minor effect on the fragments' perihelion time, the *SOHO* sungrazer population has no direct relationship to the debris associated with the 20th century cluster, which must have arrived nearly simultaneously with the bright comets, i.e., mostly in the 1960s. Instead, the *SOHO* population offers information on the debris that according to equation (8) was in 1106 released into orbits with separation velocities $V_R(t_{\rm frg})$ of approximately +2 m s^{-1} relative to the protofragment responsible for the cluster in the 1880s. The fact that no truly bright sungrazer has so far been detected in the past 11 years of the *SOHO* spacecraft's operations indicates that no fragment in this range of separation velocities was apparently massive enough to survive the relentless cascading fragmentation process during the entire revolution about the Sun.

This conclusion indicates that the *SOHO* sungrazer population represents a sample of a small section of the sheath of material in the train (Fig. 1) located between two protofragments, the products of one of which are still on their way to the Sun to

[1] See http://ares.nrl.navy.mil/sungrazer.

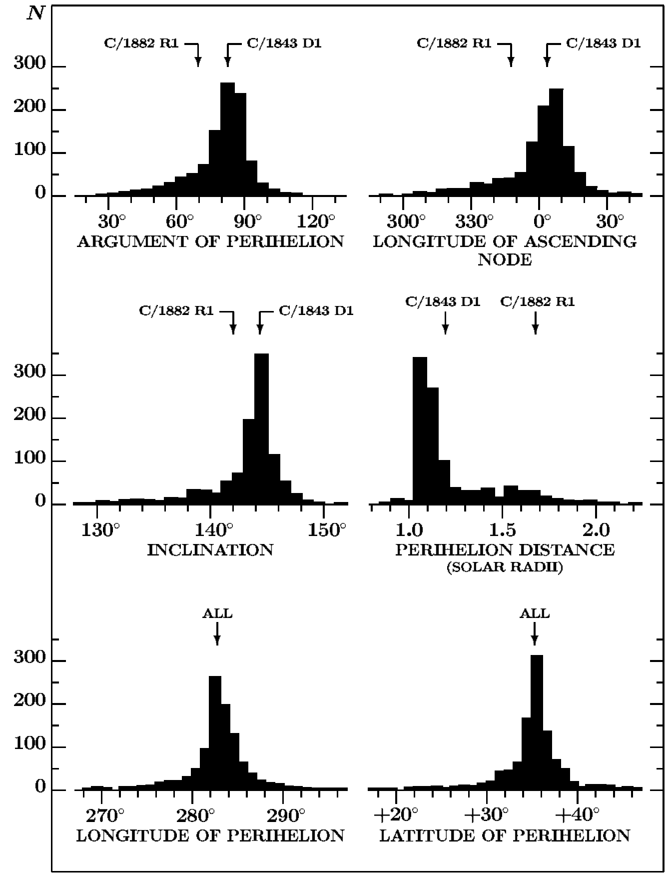

Fig. 6.—Histograms of the orbital distribution of 1000 *SOHO* sungrazers in the argument of perihelion, the longitude of the ascending node, the inclination, the perihelion distance, and the longitude and latitude of perihelion. For comparison, we show the values for C/1843 D1 and C/1882 R1.

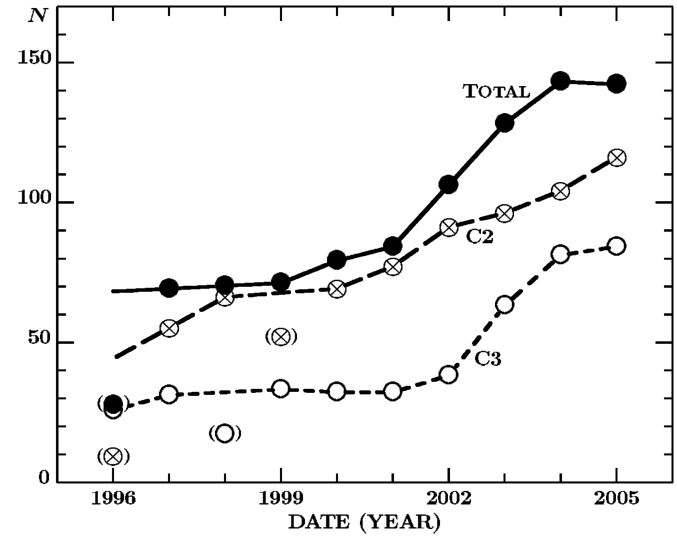

Fig. 7.—Discovery rate of the *SOHO* sungrazers as a function of date. The number of comets discovered per year, *N*, is plotted separately for the coronagraphs C2 and C3 and for the total. The parenthesized data points indicate the results affected by major data acquisition gaps. The statistics for 2006 are not yet known at the time of writing. Rapid increase in the discovery rate since 2001 is noticed.

arrive later during this century. The temporal distribution of the *SOHO* sungrazers suggests that the sampled part of the sheath is probably from its central filament, where the debris should be more sizable than at "off-axis" locations.

Variations in the discovery rate of the population of 1000 *SOHO* sungrazers detected between 1996 and 2006 can be extracted from their annual counts: both from their total and from the separate samples acquired with the C2 and C3 coronagraphs (for a reference, see footnote 1). Since many *SOHO* sungrazers are observed with both C2 and C3, the sum of the counts from C2 and C3 always exceeds the total. The integration over 1 yr is necessary because especially the C2 discovery rate is subjected to strong seasonal variations, caused by an interplay between the geometrical conditions and the comets' fading below 10 $R_\odot$ (Biesecker et al. 2002; Sekanina 2003). The C3 discovery rate is subjected to different and less pronounced fluctuations during each year, so by examining the C2 and C3 rates in parallel we can estimate the degree to which the seasonal effects have been eliminated. From plots of the number of daily images taken with the C2 and C3 coronagraphs[2] it is apparent that in 1996 the C2 temporal coverage was very incomplete, while the imaging through C3 had a lower rate than in the subsequent years, yet with no major gaps.

Significant biases of the *SOHO* sungrazer count rates were introduced by the "events" (major problems with the spacecraft persisting over extended periods of time). The worst gap, lasting for more than 3 months before the data acquisition resumed, was a loss of telemetry in late June of 1998. Fortunately, this essentially had no effect on the C2 discovery rate because the data from other years indicate that very few Kreutz sungrazers are in fact observed with C2 between the end of June and mid-October. However, the C3 rate was affected substantially.

The second event, a failure of the last gyroscope in December of the same year, resulting in a data gap of about 6 weeks, did not influence measurably the C3 rate in 1999 (only at a ~10% level or so), but four short periods (totaling 16 days) of no or almost no C2 data in mid-November through mid-December of 1999, near the time of a C2 peak rate, did affect that year's C2 statistics.

Various housekeeping and maintenance interruptions not exceeding a few days are found to have no major effect on the annual C2 or C3 rates. Similarly, there is no systematic personal bias; the art of *SOHO* comet hunting is a highly competitive affair, with multiple detections often reported. The images remain on the internet and the fact that the rate of archival discoveries has dropped rapidly in recent years suggests that the search is virtually complete. There is a file of *SOHO* X comets, observed (mostly with C2) too imperfectly to allow an orbit determination. However, their rate is low and fairly constant, ranging in the 1996–2006 period from 2 to 9 yr^{-1} and averaging $5.1 \pm 2.7\ \mathrm{yr}^{-1}$. Not all X comets are necessarily Kreutz sungrazers.

The annual averages of the *SOHO* sungrazer discovery rate for 1996–2005 are displayed in Figure 7. Instead of attempting to apply corrections to the few bad data points, we parenthesize and ignore them. The most important result is the tendency for an increase with time in the discovery rate of the *SOHO* sungrazers, which is detected in all three sets and may be an early warning of another cluster of bright sungrazers approaching the Sun in the coming decades, as already mentioned. It appears that it is only a matter of time before a brilliant sungrazer is going to lighten our skies once again.

Looking for more subtle effects in Figure 7, we note that the total number appears to have leveled off in 2004–2005, and this seems to be also supported by the statistics in 2006 (still incomplete at the time of writing). The C3 curve is most interesting, as it shows a steplike pattern, with a nearly constant rate between 1996

[2] See http://lasco-www.nrl.navy.mil/daily_statistics/daily_stat.html.

TABLE 10

Computed Sets of Orbital Elements for Progenitor Comet's Past Returns to the Sun (Equniox J2000.0)

			Osculating Orbital Element						
Scenario	Return to Sun	Osculation Epoch[a] (ET)	T[a] (ET)	ω (deg)	Ω (deg)	i (deg)	q (AU)	e	P (yr)
A..................................	1[b]	1811 Jan 8.0	1811 Jan 19.6	84.91	6.21	144.49	0.00522	0.9999354	727
	0[c]	1106 Jan 17.0	1106 Jan 26.5	84.73	5.85	144.54	0.00537	0.9999328	715
	−1[c]	423 Feb 12.0	423 Feb 7.5	82.12	2.65	144.18	0.00515	0.9999379	755
	−2	BCE 308 Oct 6.0	BCE 308 Oct 3.7	81.36	1.60	144.16	0.00538	0.9999339	735
	−3	BCE 999 Oct 30.0	BCE 999 Nov 10.2	80.63	0.60	143.96	0.00574	0.9999273	702
	−4	BCE 1623 Oct 26.0	BCE 1623 Nov 10.1	77.98	357.45	143.67	0.00639	0.9999152	653
	−5	BCE 2264 Apr 14.0	BCE 2264 Apr 6.8	78.48	357.86	143.74	0.00635	0.9999153	649
B..................................	1[b]	1764 Sep 10.0	1764 Sep 22.2	87.70	9.58	144.55	0.00516	0.9999337	687
	0[c]	1106 Jan 17.0	1106 Jan 26.5	84.73	5.85	144.54	0.00537	0.9999297	668
	−1[c]	467 Feb 21.0	467 Feb 1.5	86.25	7.62	144.62	0.00520	0.9999330	684
	−2[d]	BCE 214 Jan 22.0	BCE 214 Jan 3.7	85.72	6.82	144.56	0.00515	0.9999340	689
	−3	BCE 841 Jun 2.0	BCE 841 Jun 10.7	86.01	7.12	144.58	0.00583	0.9999209	632
	−4	BCE 1444 Aug 27.0	BCE 1444 Aug 10.5	87.67	9.08	144.62	0.00572	0.9999234	644
	−5	BCE 2086 Nov 26.0	BCE 2086 Dec 2.2	87.41	8.57	144.65	0.00554	0.9999269	660

[a] Old style except for 1811 and 1764.
[b] Applies to the center of mass of the fragmented progenitor comet.
[c] Comet listed in historical records.
[d] Listed in historical records as a probable comet (Ho 1962).

and 2002, followed by a rapid rise between 2002 and 2004, and another plateau, with a rate more than twice that of the pre-2002 period. This may suggest a nonuniform distribution of the minicomets along the filament.

Analysis of the *SOHO* sungrazer population shows that its great majority moves in orbits much more similar to C/1843 D1 than to C/1882 R1. Our inspection of diagnostic observational evidence thus leads to a conclusion that both the majority of sizable products of the protofragments (bright sungrazers) and much of the available debris located in between the protofragments (*SOHO* sungrazers) move in subgroup I–type orbits. We therefore find that if X/1106 C1 was a parent comet of most of the observed members of the Kreutz system, it was likely to move in an orbit of subgroup I, which we approximate with that of C/1843 D1 as this group's representative object.

7. FRAGMENTATION SCENARIOS A AND B

To accommodate the 17th century and earlier clusters of the Kreutz system, we now search for fragmentation scenarios, each restricted by a historical comet that could be a plausible candidate for X/1106 C1 at its previous return to the Sun. Besides the orbital similarity with C/1843 D1, our only constraint on the starting conditions is the orbital period. It should be shorter than 776 yr, the difference between 1106 and 1882, by approximately a century. The parent comet's center of mass is thus required to have passed through perihelion about midway between the late 17th century and the late 19th century clusters, that is, in either the second half of the 18th century or the early 19th century. Orbital linkage of X/1106 C1 with a sungrazer in the 5th century can "anchor" such scenarios. Hasegawa & Nakano (2001) do in fact find a potential sungrazer in 423, very close to the critical time, with no other candidate for nearly 200 yr on either side. The comet of 423 was at perihelion most probably on February 7. Orbital linkage of the comets of 1106 and 423, briefly described below, defines our *scenario A*.

Inspecting England's (2002) list of sungrazer suspects, we find another candidate, the comet of 467, which in England's opinion had equally good sungrazer credentials as the comet of 423 (rank 5). The comet of 467 passed through perihelion in early February, and its orbital linkage with X/1106 C1 defines our *scenario B*. The absence of this object on Hasegawa & Nakano's (2001) list of possible Kreutz comets can perhaps be explained by fewer details available on its path across the sky, compared to the comet of 423, and by the fact that the 467 reports came mostly from Byzantine and other European chronicles, while Hasegawa & Nakano (2001) used Far Eastern sources. There are no other comets ranked 5 or higher by England between the years 133 and 852. Strom (2002) offers no sun-comet between AD 15 and 1539, so the two 5th century comets are the only two candidates to consider.

We began by integrating the orbit of C/1843 D1 back to 1106, adjusting its eccentricity to yield Hasegawa & Nakano's (2001) date for the perihelion time of X/1106 C1, January 26. The orbital elements were referred to a 40 day standard osculation epoch nearest the 1106 perihelion time. This step is common to both scenarios.

Next, the eccentricity was adjusted again, in order that integrating the orbit from 1106 back to the 5th century fit the perihelion time of the comet of 423 in scenario A and the comet of 467 in scenario B. This integration was then extended further into the past to provide a brief history of the orbital evolution in both cases. The corresponding sets of elements are listed in Table 10. In scenario A, we were unable to identify the predicted pre-423 returns with any historical comet. In scenario B, however, the comet of 1106 and 467 could be identified with the comet of 214 BCE, if appearing in January of that year. Unfortunately, the Chinese historical record mentioning this comet gives only the year of observation (Ho 1962).

Had the parent comet not fragmented in 1106, it would have returned to the Sun in 1811 in scenario A, but in 1764 in scenario B. The approximate radial separation velocity V_R necessary for a protofragment to arrive at perihelion at any time T can be expressed similarly to that for the two-superfragment model, given by equation (8). For scenario A we find

$$V_R(t_{\rm frg},\, T) = 1.957\tau - 0.2419\tau^2 + 0.02517\tau^3, \qquad (9)$$

where we have $\tau = (T - 1811.05)/100$, 1811.05 being the perihelion time of the parent comet's center of mass. For any T, the

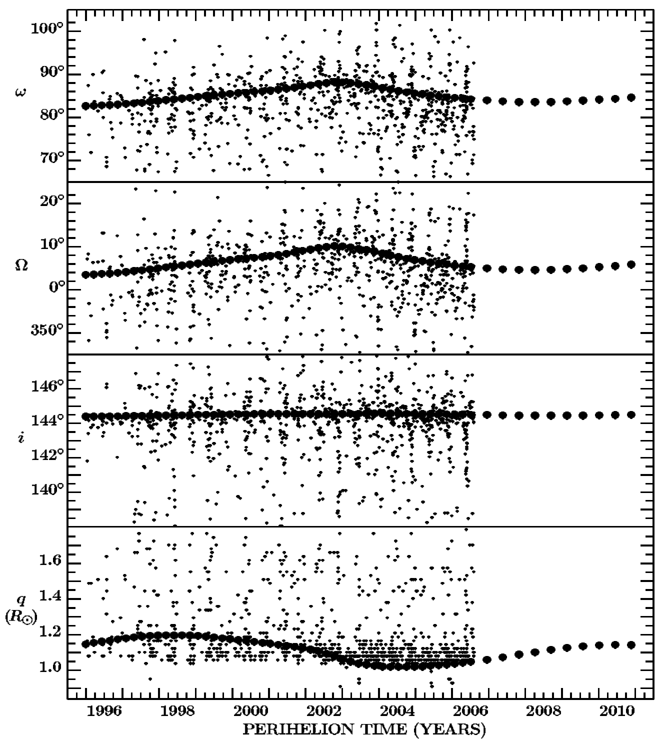

FIG. 8.—Comparison of the orbital distribution of 1000 *SOHO* sungrazers from Fig. 5 (*dots*) with the predicted values (*filled circles*) of the orbital elements derived from scenario B for the radial separation velocities from X/1106 C1 between +3.92 (beginning of 1996) and +4.12 m s^{-1} (end of 2010). The predicted orbital elements were calculated from the relevant data in Table 10 for the standard 80 day osculation epochs in 1996–2006 and for the standard 160 day osculation epochs between 2006 and 2010.

equivalent value of the transverse separation velocity is equal to $V_T(t_{\rm frg}) = 1.40 V_R(t_{\rm frg})$, whereas the equivalent separation velocity along the orbital velocity vector amounts to $V^*_{\rm sep}(t_{\rm frg}) = 0.81 V_R(t_{\rm frg})$.

For scenario B,

$$V_R(t_{\rm frg}, T) = 2.199\tau - 0.2946\tau^2 + 0.03052\tau^3, \quad (10)$$

where we now have $\tau = (T - 1764.72)/100$, with the velocity relations $V_T(t_{\rm frg}) = 1.42 V_R(t_{\rm frg})$ and $V^*_{\rm sep}(t_{\rm frg}) = 0.82 V_R(t_{\rm frg})$.

The radial separation velocities for the sungrazer clusters in scenarios A and B are listed in the last two columns of Table 9. Scenario A requires an excessive V_R (more than 5.5 m s^{-1} in absolute value; § 5) only for the 1564–1588 cluster, whose existence may be questionable, while scenario B fits all eight clusters with acceptable values of V_R. The population of *SOHO* sungrazers is accounted for by a narrow range of V_R near +3 m s^{-1} relative to the center of mass in scenario A and about +4 m s^{-1} in scenario B. The adopted limit on V_R allows future clusters to appear up to the year 2200 in scenario A and up to 2120 in scenario B.

The temporal variations in the orbital elements of the 1000 *SOHO* comets from Figure 5 are compared in Figure 8 with the orbits predicted from scenario B with the radial separation velocities from X/1106 C1 between +3.92 and +4.12 m s^{-1} relative to the center of mass for perihelion times between the beginning of 1996 and the end of 2010. We note that there is a good degree of correspondence in the trends between the maximum data point concentrations in Figure 8 and the predicted angular elements, in the sense that, in ω and Ω, they first increase until about the beginning of 2003, then decrease, while remaining essentially constant in i. In q, which is rather poorly determined for the *SOHO* sungrazers (see § 6.4), the maximum data point concentration and the predicted value both appear to remain below 1.2 $R_\odot$ at all times.

8. DISCUSSION AND CONCLUSIONS

The primary purpose of this paper has been to continue our exploration of the evolution of the Kreutz system. The fragmentation hierarchy of eight bright sungrazers of the 19th and 20th centuries presented in Paper I has now been supplemented by a conceptual development of fundamental methods and computational tools that, drawing in part from similarities with the defunct comet Shoemaker-Levy 9, allow us to broaden the scope of investigation and examine cascading fragmentation scenarios (1) for up to about 30 bright comets, the Kreutz system's definite and/or potential members, which make up eight clusters spanning the period from 1564 to 1970; and (2) for 1000 *SOHO* minisungrazers observed between 1996 and 2006. The developed methods and tools can also be used to examine the origin of other members of the Kreutz system, of various sizes and arriving at the Sun at various times, including the Solwind and *SMM* sungrazers of the 1980s, as well as sungrazers that are expected to arrive at the Sun during the 21st century and possibly beyond.

It should be emphasized that we deal with not only a major expansion but also a modification of the two-superfragment model. While we assumed in Paper I that comet X/1106 C1, a prominent historical object and almost certainly a sungrazer, moved in a subgroup II–type orbit, we now prefer its orbit to be subgroup I. We present arguments for this conclusion, which is the basis for two competing scenarios. The comets C/1882 R1 and C/1965 S1 are in this case fragments of another parent comet, which shared the common progenitor with X/1106 C1 and arrived at the Sun within several years of X/1106 C1, but remained unreported in historical sources possibly because it reached perihelion during a "wrong" season (between late May and early August).

The good fit to the orbital distribution of the population of 1000 *SOHO* sungrazers by the predicted subgroup I–type orbit of X/1106 C1 in Figure 8 is encouraging and represents a major step forward in our effort to document the role of the process of cascading fragmentation in the Kreutz system's orbital database currently available. It is expected that a further extension of this analysis in the future will provide ever tighter constraints on the fragmentation models.

The proposed fragmentation scenarios A and B are the starting points in our efforts to introduce major new ideas on the Kreutz system evolution. In this context, the importance of the two 5th century comets, 423 and 467, is greatly increased, and they both imply the progenitor with an orbital period of less than 700 yr, shorter than assumed in previous hypotheses. Also new is our consideration of a subgroup I–type orbit for comet X/1106 C1. While our assumptions entail generally favorable consequences, it is not possible to accommodate the frequent conjecture in the literature that the famous comet of 372 BCE was the sungrazers' quintessential progenitor. This comet has often been associated with the philosopher Aristotle, who besides the historian Callisthenes of Olynthus and others described it in his writings. However, Aristotle was only 12 years old at the time, while Callisthenes was born more than 10 years after the comet had appeared. Thus, their reports should not be considered eyewitness accounts. Mentioned in connection with the major earthquake in Achaea and the ensuing destruction of the cities of Helice and Buris, the comet was looked up to as a bad omen, a circumstance that aided its perceived importance. Interestingly, the comet is not listed in any Chinese chronicles or other contemporary or nearly contemporary sources from outside Greece. Endlessly copied and recopied in both the technical and outreach literature is a particularly controversial statement, attributed to Ephorus of Cyme, who wrote that apparently this same comet had split up into two stars (Barrett 1978). The professional reputation of Ephorus appears unfortunately to have been less than impeccable, at least in some quarters, judging from Seneca's comments (Barrett 1978). In any case, nowadays we know that comet splitting cannot be observed. One can detect only its product, the duplicity or multiplicity of the initially single nucleus, and all such detections have always been made telescopically. For split sungrazers, such as C/1882 R1, secondary nuclei are too faint and much too close to one another to be resolved with the naked eye. To sum it up, the presented arguments suggest that the degree of prominence and historical significance of the comet of 372 BCE may very well be overrated, and its Kreutz system credentials are by no means certain. Pingré's (1783) crude orbital elements for the comet, reproduced by Kronk (1999, p. 4), yield a perihelion direction that is more than 90° away from both the standard line of apsides of the Kreutz system and the predicted values for the proposed progenitors in Table 10. We conclude that the quality of a hypothesis for the Kreutz system evolution is not harmed if the comet of 372 BCE does not fit in.

The preliminary version of the present paper, commented on by Marsden (2005), differed in one aspect significantly from the current, final version. The difference consists in that the pair C/1882 R1 and C/1965 S1 was not incorporated into the preliminary version, which yielded 900 yr for the age of the Kreutz system and identified the progenitor with X/1106 C1. Since all fragments were released into subgroup I orbits in 1106, C/1882 R1 and C/1965 S1 could not separate from their parent within 1 AU of the Sun and move in subgroup II orbits. Proposing now to combine a scenario for cascading fragmentation with a new version of the two-superfragment model, we conceptually resolve this problem by virtue of introducing a second parent comet besides X/1106 C1.

If we insist on limiting *all* separation velocities to values not exceeding 5.5 m s^{-1}, an upper limit established in § 5 from information on C/1882 R1 and C/1965 S1, the fragmentation scenario for their birth derived in Sekanina & Chodas (2002a) should be refined. It is conceivable that this pair of sungrazers originated from a two-step process. The first, tidally driven fragmentation event, involving a low separation velocity of $\sim$1–2 m s^{-1} (Fig. 3), should account for the temporal separation of 83 yr between the two comets, whereas a subsequent nontidal breakup of the smaller mass with a similarly low separation velocity should explain slight changes in the other orbital elements of the resulting fragment. The assumed continuing cascading fragmentation of the other piece or pieces, of no interest to the hypothesis for the pair birth, should ensure that only one secondary fragment is left to stand as C/1965 S1. The uncomfortably high separation velocity of 7 m s^{-1} in the forced single-event solution (Sekanina & Chodas 2002a) is thus disposed of.

The same two-step process should also be applied to refine the birth scenarios of some of the other bright sungrazers that required separation velocities as high as 10 m s^{-1} in the two-superfragment scheme in Paper I, including notably C/1963 R1 and C/1945 X1. Such two-step scenarios might change the fragmentation hierarchy of the bright sungrazers of the 19th and 20th century clusters to the extent that the relationships among the comets may no longer resemble those shown in Figure 2 of Paper I.

The details of the primary nontidal breakup of the progenitor into the two parent comets in the proposed scenarios A and B can only be guessed. The heliocentric distance of $\sim$50 AU derived in Paper I for the fragmentation event allows us to estimate that it occurred about AD 390–395 in scenario A and about AD 435–440 in scenario B. There obviously is some uncertainty involved in the progenitor's orbital period, estimated at a few years. Hence, the predicted perihelion times in Table 10 prior to the 5th century could be correspondingly uncertain, and so could be the progenitor's identity with the comet of 214 BCE (taken from Ho 1962) in scenario B.

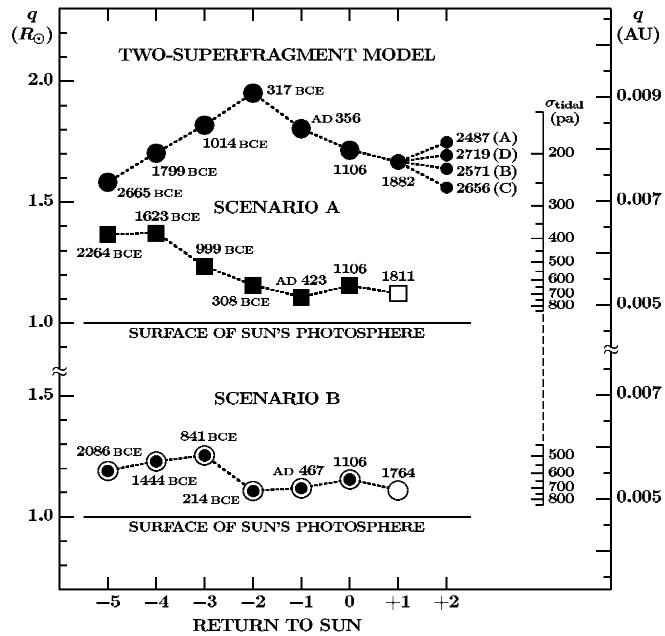

Fig. 9.—Evolution of the progenitor's perihelion distance for the two-superfragment model from Paper I and for scenarios A and B. Also shown are the corresponding peak tensile stress the nucleus was subjected to (see eq. [11]) and the data for the four nuclear condensations A–D of C/1882 R1 in the two-superfragment model from Paper I.

Although we gave reasons in § 6.4 for attributing the orbit of X/1106 C1 to subgroup I, the ambiguity of the two parent comets as the first-generation fragments of the progenitor really persists. If we had at least vague information on the second parent sungrazer in the early 12th century, we could experiment with various orbital combinations of X/1106 C1 and this second object vis-à-vis their relation to subgroups I and II. In the absence of any such information, we needed to "borrow" X/1106 C1 to develop the two scenarios. Interestingly, in his very different conceptual model for the Kreutz system, Marsden (1989) also considered the existence of a second sungrazer at about the same time, between 1102 and 1114.

While subject to some uncertainties, the results, in Table 10, of our orbital integration back in time allow us to examine the temporal variations in the progenitor's predicted past perihelion distance and estimate peak tidal stresses that the object was subjected to at its perihelion points. Figure 9 compares the two-superfragment model from Paper I with the proposed scenarios. The peak tidal stress by the Sun (in pascals) on the nucleus of a comet, whose bulk density is ρ (in g cm^{-3}) and characteristic dimension for tidally driven fragmentation is $\Re$ (in kilometers), can be expressed by a formula

$$\sigma_{\rm tidal} = 197\rho\Re^2(R_\odot/q)^3, \qquad (11)$$

where $R_\odot$ is the solar radius and q is the perihelion distance. For a comet of an initial diameter D splitting into $N_{\rm frg}$ major fragments, $\Re$ can be approximated by $\Re \simeq D/N_{\rm frg}$. The scale for the peak tidal stress in Figure 9, derived with an assumed value of

$\rho\Re^2 = 5$ km^2 g cm^{-3}, shows that the net peak tidal stress for the sungrazers is on the same order of magnitude as that found for D/Shoemaker-Levy 9 (380 Pa; Sekanina 1996) and as the tensile strength predicted for cometary nuclei by Greenberg et al. (1995) from molecular interactions at the contact surfaces of interstellar dust grain aggregates (270 Pa). We note that the assumed value of $\Re$ is 5 km for a density of 0.2 g cm^{-3} and 2.5 km for a density of 0.8 g cm^{-3}.

Even though the progenitor's breakup was clearly nontidal in nature, the tidal stresses the object had apparently withstood during preceding returns to the Sun may have contributed to the fragmentation event by inflicting cracks in the object's interior. Interestingly, Figure 9 shows that the predicted perihelion distance was smaller (and the peak tidal stress higher) in both 308 BCE (scenario A) and 214 BCE (scenario B) compared with that during the previous returns between the 9th and 23rd centuries BCE. This circumstance favors the scenarios A and B as it makes the progenitor's breakup more likely in the estimated period of the late 4th century or the early 5th century AD, on the way to perihelion in 423 or 467, than at other times during the preceding two to three millennia.

It is also noted from Figure 9 that the two-superfragment model developed in Paper I on the assumption of a subgroup II–type orbit for X/1106 C1 shows perihelion distance variations incongruous with the expected trend of a Kozai cycle, peaking at 317 BCE and therefore inconsistent with a hypothesis of cracks generated in the interior of the progenitor during the perihelion passage just preceding the primary nontidal fragmentation event in the year 326. The unexpected perihelion distance trend appears to be due to the effect of Jovian indirect perturbations, which modifies the overall trend of the Kozai cycle; the comet's perihelion distance is a function of Jupiter's position in its orbit at the time the comet is at perihelion. One could go as far as to suggest that this unwelcome perihelion distance peak in 317 BCE makes it unlikely for X/1106 C1 to move in a subgroup II–type orbit.

A major property of cascading fragmentation is a very limited survival of clusters, which have a tendency to disappear after one revolution about the Sun. If the four most persisting condensations of C/1882 R1 should survive one revolution, Figure 9 shows that they would return to the Sun (with an uncertainty of several years) in 2487 (condensation A), 2571 (B), 2656 (C), and 2719 (D), that is, at intervals of, respectively, 84, 85, and 63 yr. As their fragmentation will have in all probability continued before they arrive, the observed products will be clusters of comets appearing at times approximately centered on the above dates. If C/1963 R1 is observed again during its next return, predicted for 2753, it could easily be mistaken for a trailing member of the cluster of 2719. A similar problem may also be posed by C/1965 S1 and perhaps by some other members of the 19th and 20th century clusters.

The possible simultaneous existence of products from two (or more) tidally initiated fragmentation branches of sungrazers has further major ramifications. If products of an earlier branch can be confused with products of a subsequent branch in the future, the same problem may have occurred in the past. Even sungrazers near the middle of clusters may be strayed members of another branch, or branches, not related to the products of the same parent comet. This is particularly the case when the progenitor splits at a large heliocentric distance and the first-generation fragments reach their perihelion nearly simultaneously.

It should further be emphasized that the parallel existence of products from consecutive branches of any tidally driven fragmentation process is restricted in practice by its high degree of wastefulness; the disintegration rate is too high for the process to perpetuate itself many times. The long-term clustering of bright sungrazers is a fitting illustration of the enormous rate of mass destruction involved. Unless the progenitor's initial dimensions were hundreds of kilometers (with a corresponding mass greatly exceeding 10^{21} g), the process could hardly pass through more than two or three cycles, as each time it was drastically scaled down compared to the previous cycle's level.

Because of the extremely limited orbital database and the enormous number of possible evolutionary paths, it is wholly impossible to determine the unique, rigorous solution to the problem of dynamical evolution of the Kreutz system. The only reasonable approach to demonstrate the usefulness of the developed methods and computational tools and to show the role of the two-superfragment model in the context of the process of cascading fragmentation is to present at least one detailed self-consistent orbital scenario involving all the bright sungrazers and the population of the *SOHO* minicomets. We will make an effort to present such a scheme in the forthcoming Part III of this series. This attempt will use the fact we learned from our experimentation (Sekanina & Chodas 2002a), namely, that the orbital evolution solutions are, in general, remarkably insensitive to the temporal distribution of events in the fragmentation sequence.

The future of observing the Kreutz sungrazer system looks bright. Provided that the *SOHO* spacecraft remains healthy, its operations will continue for at least 3 more years, given an ESA's recent decision to fund the mission through the end of 2009.[3] Also encouraging is NASA's launch of the twin *STEREO* space observatories on 2006 October 25 and the initial successful imaging tests of the SECCHI SCIP-A and SCIP-B instruments 40 and 50 days later.[4] During 2007, these spacecraft are expected to usher in a new era of near-Sun comet hunting. A simultaneous operation, together with *SOHO*, of three spacecraft will undoubtedly prove revolutionary. There are also signs that another cluster of bright Kreutz system comets is on its way to the Sun in the coming decades, with the earliest objects expected to arrive perhaps as soon as several years from now. During the next few decades, mankind should once again witness spectacular heavenly shows like that in 1965.

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[3] See http://www.esa.int/esaCP/SEMSVJ9ATME_index_0.html.
[4] See http://stereo-ssc.nascom.nasa.gov/new.shtml.

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